# An $U(1)$ anomaly free for three families

S. Mantilla, R. Martínez, F. Ochoa
Departamento de Física
Universidad Nacional de Colombia,
Bogotá
SILAFAE 2016, November 14-18, 2016

November 16, 2016

## Introduction

- Models with extra $U(1)$ symmetry are studied as extensions of the Standard Model (SM). There are many motivations to consider this kind of models.
- In supersymmetric extensions, an additional $U(1)$ factor may provides a mechanism to generate the $\mu_{eff}$ term through the addition of a scalar singlet.
- Non-supersymmetric extensions give rise to a variety of models like left-right symmetric models, B-L, flip grand unified models, $E_6$, $SO(10)$, etc, which involve theoretical and phenomenological aspects:

1. Flavor physics, Flavor Changing,
2. Neutrino physics,
3. Dark matter,
4. Aditional Higgs,
5. Neutral currents, $Z'_\mu$

- They provide hints to expalin the SM mass hierarchy problem, where top quark acquires mass at the EW scale and the other fermions exhibit different low mass values.
- Introduced right handed sterile neutrinos to explain the masses and mixing of the active neutrinos.
- These extensions have Two Higgs Doublet Models in the low energy limit, where two scalar doublets $\phi_1$ and $\phi_2$ are introduced in order to generate the appropriate Yukawa couplings that provide masses to all fermions.

- A singlet scalar field $\chi$ is introduced to break $U(1)$ symmetry and to give masses to exotic particles beyong the SM.
- A singlet scalar field $\sigma$ can be considered to study dark matter problem.
- In addition of a new neutral gauge boson $Z'$, extended fermion spectrum is necessary in order to obtain an anomaly-free theory. LHC collider pushes a lower mass bound to $Z'$ of the order of 3 TeV.
- Since the new symmetry introduces an additional gauge boson, there arise new couplings that induce nontrivial triangle anomalies
- There are 6 possible combinations $Tr[T'_{SM}T_{SM}T_X] = 0$

## Quiral anomalies

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$$

$$\begin{aligned}
[SU(3)_c]^2\, U(1)_X &\rightarrow A_1 = \sum_Q X_{Q_L} - \sum_Q X_{Q_R} \\
[SU(2)_L]^2\, U(1)_X &\rightarrow A_2 = \sum_\ell X_{\ell_L} + 3 \sum_Q X_{Q_L}, \\
[U(1)_Y]^2\, U(1)_X &\rightarrow A_3 = \sum_{\ell,Q} \left[ Y^2_{\ell_L} X_{\ell_L} + 3\, Y^2_{Q_L} X_{Q_L} \right] \\
&- \sum_{\ell,Q} \left[ Y^2_{\ell_R} X_{\ell_R} + 3\, Y^2_{Q_R} X_{Q_R} \right]
\end{aligned}$$

where the sums in $Q$ run over all the quarks $(u^i, d^i, T, J^n)$, and $\ell$ runs over all leptons $e^i, \nu^i_L, \nu^i_R, N^i_R$.

$$
\begin{aligned}
U(1)_Y\,[U(1)_X]^2 \quad &\rightarrow \quad A_4 = \sum_{\ell,Q}\left[Y_{\ell_L}X_{\ell_L}^2 + 3Y_{Q_L}X_{Q_L}^2\right] \\
&- \quad \sum_{\ell,Q}\left[Y_{\ell_R}X_{\ell_R}^2 + 3Y_{Q_R}X_{Q_R}^2\right] \\
[U(1)_X]^3 \quad &\rightarrow \quad A_5 = \sum_{\ell,Q}\left[X_{\ell_L}^3 + 3X_{Q_L}^3\right] - \sum_{\ell,Q}\left[X_{\ell_R}^3 + 3X_{Q_R}^3\right] \\
[Grav]^2 \otimes U(1)_X \quad &\rightarrow \quad A_6 = \sum_{\ell,Q}\left[X_{\ell_L} + 3X_{Q_L}\right] \\
&- \quad \sum_{\ell,Q}\left[X_{\ell_R} + 3X_{Q_R}\right]
\end{aligned}
$$

There are some solutions to anomaly equations. But we consider one of them which can explain mass herarchy problem. The second column $U(1)_X$ are the values of the new quantum number.

## Particle contents: Fermions

| Quarks | $X$ | Leptons | $X$ |
|---|---|---|---|
| SM Fermionic Isospin Doublets | | | |
| $q_L^{u,c} = \begin{pmatrix} u^{u,c} \\ d^{u,c} \end{pmatrix}_L$ | 0 | $\ell_L^{e,\mu} = \begin{pmatrix} \nu^{e,\mu} \\ e^{e,\mu} \end{pmatrix}_L$ | 0 |
| $q_L^t = \begin{pmatrix} u^t \\ d^t \end{pmatrix}_L$ | 1/3 | $\ell_L^\tau = \begin{pmatrix} \nu^\tau \\ e^\tau \end{pmatrix}_L$ | -1 |
| SM Fermionic Isospin Singlets | | | |
| $u_R^{u,c,t}$ | $2/3$ | $e_R^{e,\tau}$ | $-4/3$ |
| $d_R^{u,c,t}$ | $-1/3$ | $e_R^\mu$ | $-1/3$ |
| Non-SM Quarks | | Non-SM Neutrinos | |
| $T_L$ | $1/3$ | $\nu_R^{e,\mu,\tau}$ | $1/3$ |
| $T_R$ | $2/3$ | $N_R^{e,\mu,\tau}$ | 0 |
| $J_L^{1,2}$ | 0 | | |
| $J_R^{1,2}$ | -1/3 | | |

- The $U(1)_X$ symmetry is non-universal in the left-handed SM quark sector.
- The SM leptons are also non universal.
- The right handed quarks are universal, but the right handed leptons are non universal.
- The three extra singlets $T$ and $J^n$ are new up- and down-like quarks, respectively, where $n = 1, 2$.
- We include new neutrinos $(\nu_R^i)$ and $N_R^i$ which may generate see-saw neutrino masses in order to obtain a realistic model compatible with oscillation data.
- $(\nu_R^i)$ and $\nu_L$ generate Dirac masses and $N_R^i$ produce Majorana mass terms.

## Particle contents: Bosons

| Scalar bosons | $X$ |
|---|---|
| **Higgs Doublets** | |
| $\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{h_1+v_1+i\eta_1}{\sqrt{2}} \end{pmatrix}$ | 2/3 |
| $\phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{h_2+v_2+i\eta_2}{\sqrt{2}} \end{pmatrix}$ | 1/3 |
| **Higgs Singlets** | |
| $\chi = \frac{1}{\sqrt{2}} \left(\xi_\chi + v_\chi + i\zeta_\chi\right)$ | -1/3 |
| $\sigma$ | -1/3 |

**Table:** Non-universal $X$ quantum number for Higgs fields.

The scalar doublet $\phi_1$ also has a nontrivial charge $X$.

- The spectrum includes an additional scalar doublet $\phi_2$ identical to $\phi_1$ under $G_{SM}$ but with different $U(1)_X$ charges, where the electroweak scale is related to the VEVs by $\nu = \sqrt{\nu_1^2 + \nu_2^2}$.
- An extra scalar singlet $\chi$ with VEV $\nu_\chi$ is required to produce the symmetry breaking of the $U(1)_X$ symmetry. We assume that it happens at a large scale $\nu_\chi \gg \nu$.
- Another scalar singlet $\sigma$ can be introduced as a dark matter candidate.
- We define the weak hypercharge $Y$ as usual, where the electric charge is defined by the Gell-Mann-Nishijima relation:

$$Q = T_{3L} + \frac{Y}{2}$$

with $T_{3L}$ the isospin defined for left- and right-handed fermions.

## The Higgs Potential

The renormalizable and $G_{SM} \times U(1)_X$ invariant potential is

$$
\begin{aligned}
V &= \mu_1^2 \phi_1^\dagger \phi_1 + \mu_2^2 \phi_2^\dagger \phi_2 + \mu_3^2 \chi^* \chi + f_2 \left( \phi_1^\dagger \phi_2 \chi + h.c. \right) \\
&+ \lambda_1 \left( \phi_1^\dagger \phi_1 \right)^2 + \lambda_2 \left( \phi_2^\dagger \phi_2 \right)^2 + \lambda_3 \left( \chi^* \chi \right)^2 \\
&+ \lambda_4 \left( \phi_1^\dagger \phi_1 \right) \left( \phi_2^\dagger \phi_2 \right) + \lambda_5 \left( \phi_1^\dagger \phi_2 \right) \left( \phi_2^\dagger \phi_1 \right) \\
&+ \lambda_6 \left( \phi_1^\dagger \phi_1 \right) \left( \chi^* \chi \right) + \lambda_7 \left( \phi_2^\dagger \phi_2 \right) \left( \chi^* \chi \right)
\end{aligned} \tag{1}
$$

When we apply the minimum conditions for each scalar VEV $\nu_i = \nu_{1,2,\chi}$, we find the square mass matrices $M_R^2$ for the real fields, $M_I^2$ for the imaginary fields and $M_C^2$ for the charged fields.

$$M_R^2 = \begin{pmatrix} 4\lambda_1\nu_1^2 - f_2\frac{\nu_2\nu_\chi}{\nu_1} & 2f_2\nu_\chi + 2(\lambda_5+\lambda_5')\nu_1\nu_2 & f_2\nu_2 + 4\lambda_6\nu_1\nu_\chi \\ * & 4\lambda_2\nu_2^2 - f_2\frac{\nu_1\nu_\chi}{\nu_2} & f_2\nu_1 + 2\lambda_7\nu_2\nu_\chi \\ * & * & 4\lambda_3\nu_\chi^2 - f_2\frac{\nu_1\nu_2}{\nu_\chi} \end{pmatrix}$$

in the basis $h_1, h_2, \xi_\chi$ CP - even,

$$M_I^2 = \begin{pmatrix} -f_2\frac{\nu_1\nu_\chi}{\nu_2} & f_2\nu_\chi & f_2\nu_1 \\ * & -f_2\frac{\nu_2\nu_\chi}{\nu_1} & -f_2\nu_2 \\ * & * & -f_2\frac{\nu_1\nu_2}{\nu_\chi} \end{pmatrix},$$

in the basis $\eta_1^0, \eta_2^0, \zeta_\chi$ CP - odd, and

$$M_C^2 = \begin{pmatrix} -f_2\frac{\nu_1\nu_\chi}{\nu_2} - \lambda_5'\nu_1^2 & f_2\nu_\chi + \lambda_5'\nu_1\nu_2 \\ * & -f_2\frac{\nu_2\nu_\chi}{\nu_1} - \lambda_5'\nu_2^2 \end{pmatrix},$$

in the basis $\phi_1^+, \phi_2^+$.

After diagonalization, we obtain the following physical spectrum and their squared masses:

$$\begin{aligned} m^2_{h_0} &\approx \nu^2 \left[\lambda_2 S^4_\beta + \lambda_1 C^4_\beta + (\lambda_5 + \lambda'_5) C^2_\beta S^2_\beta\right] \\ m^2_{H_0} &\approx \frac{2 f_2 \nu_\chi}{C_\beta S_\beta} \ , \ \ m^2_{H^0_\chi} \approx 8\lambda_3 \nu^2_\chi \end{aligned}$$

for the real sector, CP-even,

$$m^2_{A_0} = \frac{m^2_{H_0}}{2}\left[1 + C^2_\beta S^2_\beta \left(\frac{\nu}{\nu_\chi}\right)^2\right]$$

for the pseudoscalar boson, CP-odd, and

$$m^2_{H^\pm} = \frac{m^2_{H_0}}{2}\left[1 + \lambda'_5 C_\beta S_\beta \left(\frac{\nu^2}{2 f_2 \nu_\chi}\right)\right]$$

for charged Higgs bosons.

In the above expresions, we define the electroweak VEV and the angle

$$\nu = \sqrt{\nu_1^2 + \nu_2^2}\ ,\ \tan(\beta) = \frac{\nu_1}{\nu_2}.$$

In addition, we obtain two charged and two neutral would-be Goldstone bosons, which will give masses to two charged ($W^\pm$) and two neutral $Z$ and $Z'$ gauge bosons, respectively.
Symmetry breaking scheme is giving by

$$\begin{array}{rcl} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X & \longrightarrow & \nu_\chi \\ & & Z', H_0, A_0, H^\pm \\ \\ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y & \longrightarrow & \nu_1, \nu_2 \\ & & Z, W^\pm, h_0 \\ \\ SU(3)_C \otimes U(1)_Q & & \end{array}$$

## Yukawa Lagrangian

We find the Yukawa Lagrangian compatible with the $G_{SM} \times U(1)_X$ symmetry. For the Q and l sectors we find:

$$
\begin{aligned}
-\mathcal{L}_Q &= \overline{q^t_L}\left(\widetilde{\phi}_2 h^U_2\right)_{tj} U^j_R + \sum_{a=u,c} \overline{q^a_L}(\widetilde{\phi}_1 h^U_1)_{aj} U^j_R \\
&+ \overline{q^t_L}\left(\phi_1 h^D_1\right)_{tj} D^j_R + \sum_{a=u,c} \overline{q^a_L}\left(\phi_2 h^D_2\right)_{aj} D^j_R \\
&+ \overline{q^t_L}(\phi_1 h^J_1)_{tm} J^m_R + \sum_{a=u,c} \overline{q^a_L}\left(\phi_2 h^J_2\right)_{am} J^m_R \\
&+ \overline{q^t_L}\left(\widetilde{\phi}_2 h^T_2\right)_{t} T_R + \sum_{a=u,c} \overline{q^a_L}(\widetilde{\phi}_1 h^T_1)_{a} T_R \\
&+ \overline{T_L}\left(\chi^* h^U_\chi\right)_{j} U^j_R + \overline{T_L}\left(\chi^* h^T_\chi\right) T_R \\
&+ \sum \overline{J^n_L}\left(\chi h^D_\chi\right)_{nj} D^j_R + \overline{J^n_L}\left(\chi h^J_\chi\right)_{nm} J^m_R + h.c.,
\end{aligned}
$$

## Quark masses

$$
\begin{aligned}
-\mathcal{L}_l &= \sum_{i,j=e.\mu.\tau} h^{ij}_{e2}\bar{\ell}^i_L \phi_2 e^j_R + \text{h.c.} \\
&+ \sum_{i=e,\mu;j=e,\mu,\tau} h^{ij}_{\nu 2}\bar{\ell}^i_L \tilde{\phi}_2 \nu^j_R \\
&+ \sum_{i,j=e,\mu,\tau} h^{ij}_{\chi N}\nu^{i\mathrm{T}}_R \hat{C}\chi N_R + \frac{1}{2} N^{i\mathrm{T}}_R \hat{C} M^{ij}_N N^j_R + \text{h.c.}
\end{aligned}
$$

- The $U(1)_X$ symmetry distinguishes the quark family $q^t_L$ from the others two $q^{a=u,c}_L$, while the right-handed components are universal. Thus, in the absence of the Yukawa couplings, the model has the global symmetry:

$$G_{global}(h^Q = 0) = SU(2)_{q^a} \times SU(3)_{U^i} \times SU(3)_{D^i}.$$

- In particular, the $SU(2)_{q^a}$ symmetry in the left-handed sector remains in the model even after the gauge symmetry breaking. However, the experimental observation shows that this symmetry does not remain if the quark masses are taken into account.
- The extra $U(1)_X$ symmetry is not sufficient to explain the mass spectrum. Thus, we assume the existence of global symmetries, $Z_2 \times U(1)_{T_3}$.

$$\begin{aligned} U(1)_{T_3}: &\quad D^1_L \to D^1_L, \;\; D^2_L \to -D^2_L, \\ Z_2: &\quad \phi_2 \to -\phi_2, \;\; D^i_R \to -D^i_R, \;\; T_{L,R} \to -T_{L,R}. \end{aligned}$$

Thus, by requiring the discrete symmetries, the mass Lagrangian becomes:

$$
\begin{aligned}
-\langle\mathcal{L}_Q\rangle &= \sum_{i=c,t}\overline{U_L^i}\left(\nu_1 h_1^U\right)_{ij}U_R^j+\overline{D_L^3}\left(\nu_2 h_2^D\right)_{3j}D_R^j \\
&+ \overline{U_L^i}\left(\nu_2 h_2^T\right)_i T_R \\
&+ \left[\overline{D_L^1}\left(\nu_1 h_1^J\right)_{1m}+\overline{D_L^2}\left(\nu_2 h_2^J\right)_{2m}+\overline{D_L^3}\left(\nu_1 h_1^J\right)_{3m}\right]J_R^m \\
&+ \overline{T_L}\left(\nu_\chi h_\chi^T\right)T_R+\overline{J_L^n}\left(\nu_\chi h_\chi^J\right)_{nm}J_R^m+h.c.
\end{aligned}
$$

$$
\begin{aligned}
-\langle\mathcal{L}_Q\rangle &= \overline{U_L^i}(M_U)_{ij}U_R^j+\overline{D_L^i}(M_D)_{ij}D_R^j+\overline{U_L^i}(k)_iT_R+\overline{D_L^i}(s)_{im}J_R^m \\
&+ \overline{T_L}(K)_jU_R^j+\overline{T_L}(M_T)T_R+\overline{J_L^n}(S)_{nj}D_R^j+\overline{J_L^n}(M_J)_{nm}J_R^m+h.c.
\end{aligned}
$$

The extended mass matrices for up and down sectors are

$$
M'_U = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & | & \nu_2 y_1 \\ \nu_1 a_{21} & \nu_1 a_{22} & \nu_1 a_{23} & | & 0 \\ \nu_1 a_{31} & \nu_1 a_{32} & \nu_1 a_{33} & | & 0 \\ — & — & — & — & — \\ \nu_2 c_1 & \nu_2 c_2 & \nu_2 c_3 & | & \nu_\chi h_\chi^T \end{pmatrix},
$$

$$
M'_D = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & | & \nu_1 j_{11} & \nu_1 j_{12} \\ 0 & 0 & 0 & | & \nu_2 i_{21} & \nu_2 i_{22} \\ \nu_2 B_{31} & \nu_2 B_{32} & \nu_2 B_{33} & | & 0 & 0 \\ — & — & — & — & — & — \\ \nu_2 C_{11} & \nu_2 C_{12} & \nu_2 C_{13} & | & \nu_\chi k_{11} & \nu_\chi k_{12} \\ \nu_2 C_{21} & \nu_2 C_{22} & \nu_2 C_{23} & | & \nu_\chi k_{21} & \nu_\chi k_{22} \end{pmatrix}.
$$

which exhibit non-vanishing determinant, providing masses to all quarks. Due to the mixing components, the mass matrices $M'_{U,D}$ have the following propertais

The upper left $3 \times 3$ blocks:

- Exhibit three massless quarks ($m^0_{u,d,s} = 0$) and
- Three massive quarks ($m^0_{(c,t),b} \sim \nu_{1,2}$),

However $M'_{U,D}$ matrices:

- have three eigenvalues $m_{u,d,s}$ at the MeV scale,
- three $m_{c,b,t}$ at the GeV scale and
- three $m_{T,J}$ and the TeV scale.

## Specific ansatz

To explore the consequences of the above mass scheme, we assume the scenery

- Where the mixing terms are diagonal ($c_{2,3} = k_{ij} = j_{ij} = C_{ij} = 0$ for $i \neq j$, while $\nu_1 j_{11} = \nu_2 i_{22} = h_D$ and $C_{11} = C_{22} = \Gamma_D$),
- The $M_U$ sector have identical Yukawa components except the top coupling (i.e. $a_{ij} = Y_U$ for $ij \neq 33$ and $a_{33} = Y_t$);
- $M_D$ have identical components (i.e. $B_{31} = B_{32} = B_{33} = Y_D$).
  Thus, the matrices become:

$$M'_U = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & | & \nu_2 y_1 \\ \nu_1 Y_U & \nu_1 Y_U & \nu_1 Y_U & | & 0 \\ \nu_1 Y_U & \nu_1 Y_U & \nu_1 Y_t & | & 0 \\ — & — & — & — & — \\ \nu_2 c_1 & 0 & 0 & | & \nu_\chi h_\chi^T \end{pmatrix},$$

$$M'_D = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & | & h_D & 0 \\ 0 & 0 & 0 & | & 0 & h_D \\ \nu_2 Y_D & \nu_2 Y_D & \nu_2 Y_D & | & 0 & 0 \\ — & — & — & — & — & — \\ \nu_2 \Gamma_D & 0 & 0 & | & \nu_\chi k_{11} & 0 \\ 0 & \nu_2 \Gamma_D & 0 & | & 0 & \nu_\chi k_{22} \end{pmatrix}.$$

The above matrices are diagonalized through bi-unitary transformation of the form $m_Q = (\mathcal{O}_L^Q)^\dagger M'_Q \mathcal{O}_R^Q$, with $m_Q$ a diagonal matrix with real and positive values. We find for the up sector the following approximate eigenvalues:

# Eigenvalues up masses

$$
\begin{aligned}
\lambda_1^U &= m_u \approx \left(\frac{y_1 c_1 \nu_2^2}{\sqrt{2} h_\chi^T \nu_\chi}\right) = y_1 c_1 \left(\frac{\nu_2^2}{2 m_T}\right) \\
\lambda_2^U &= m_c \approx \frac{\nu_1}{2\sqrt{2}}(Y_U + Y_t)\left[1 - \sqrt{1 + 4 y_{Ut}\epsilon_{Ut}}\right] \\
\lambda_3^U &= m_t \approx \frac{\nu_1}{2\sqrt{2}}(Y_U + Y_t)\left[1 + \sqrt{1 + 4 y_{Ut}\epsilon_{Ut}}\right] \\
\lambda_4^U &= m_T \approx \frac{1}{\sqrt{2}} h_\chi^T \nu_\chi,
\end{aligned}
\tag{2}
$$

where we consider that $\nu_\chi \gg \nu_2$ and define the parameters

$$
y_{Ut} = \frac{Y_U}{Y_U + Y_t}, \qquad \epsilon_{Ut} = \frac{Y_U - Y_t}{Y_U + Y_t}. \tag{3}
$$

The parameter $\epsilon_{Ut}$ "measures" the level of asymmetry of Yukawa interactions between the top and charm quarks.

## Eigenvalue down masses

For the down sector we find:

$$
\begin{aligned}
\lambda_1^D &= m_d \approx \left(\frac{\Gamma_D h_D \nu_2 \nu_1}{\sqrt{2} k_{11} \nu_\chi}\right) = j_{12}\Gamma_D \left(\frac{\nu_1 \nu_2}{2 m_{J^1}}\right) \\
\lambda_2^D &= m_s \approx \left(\frac{\Gamma_D h_D \nu_2 \nu_1}{\sqrt{2} k_{22} \nu_\chi}\right) = j_{12}\Gamma_D \left(\frac{\nu_1 \nu_2}{2 m_{J^2}}\right) \\
\lambda_3^D &= m_b \approx \frac{1}{\sqrt{2}} Y_D \nu_2 \\
\lambda_4^D &= m_{J^1} \approx \frac{1}{\sqrt{2}} k_{11} \nu_\chi, \\
\lambda_5^D &= m_{J^2} \approx \frac{1}{\sqrt{2}} k_{22} \nu_\chi.
\end{aligned}
$$

In this case, the ratio between the masses of the down and strange quarks gives:

$$\frac{m_d}{m_s} = \frac{m_{J^2}}{m_{J^1}}.$$

Thus, the ratio between the lightest quarks is determined only by the mass splitting of the heavy quarks $J^1$ and $J^2$.
Regarding $m_u$ and $m_b$, we find that:

$$\frac{m_u}{m_b} = \left(\frac{y_1 c_1}{\sqrt{2} Y_D}\right) \frac{\nu_2}{m_T}.$$

Furthermore, the ratio between the mass of the c- and t-quark is sensible to the asymmetry parameter according to:

$$\frac{m_c}{m_t} \approx \frac{-y_{Ut}\epsilon_{Ut}}{1 + y_{Ut}\epsilon_{Ut}}. \tag{4}$$

Considering the central values, the experimental masses of the phenomenological quarks are:

$$\begin{aligned} m_u &= 2.3 \text{ MeV}, \qquad m_d = 4.8 \text{ MeV}, \qquad m_s = 95 \text{ MeV}, \\ m_c &= 1.275 \text{ GeV}, \qquad m_b = 4.65 \text{ GeV}, \qquad m_t = 173.5 \text{ GeV} \end{aligned}$$

Using the above values, leads to:

$$\begin{aligned} y_{Ut}\epsilon_{Ut} &= \frac{-m_c/m_t}{1+m_c/m_t} \approx -7,3 \times 10^{-3} \\ m_{J^1} &\approx 20\ m_{J^2} \\ \frac{\nu_2}{m_T} &\approx \left(5 \times 10^{-4}\right) \frac{\sqrt{2}Y_D}{y_1 c_1}. \end{aligned}$$

- $Y_U/Y_t \approx 0.007$ is required to fit the experimental masses.
- This ratio implies an asymmetry factor $\epsilon_{Ut} \approx -0.985$.
- For example, if $m_T \sim 1$ TeV and $\nu_2 \sim 70$ GeV we obtain $Y_D \sim 10^2 y_1 c_1$.
- Finally, we find that the large splitting between $m_d$ and $m_s$ is consequence of the existence of non-degenerated heavy massive quarks, $J^1$, $J^2$.
- To obtain the observable $m_d/m_s$ ratio we need $M_{J^2} \sim 1$ TeV and $M_{J^1} \sim 20$ TeV.
- Thus, the model predicts new quarks which can explain a mass hierarchy observed among the SM quark families.

## Lepton masses and mixing angles

- A leptonic mixing matrix is present in weak charged current which is the product of the rotation matrices of the charged leptons and neutrinos.
- The neutrino oscillation experiments have stablished that neutrinos are massive and there is lepton flavor violation.
- From the neutrino global analysis can be obtained the best allow region for these 6 parameters: $\Delta m^2_{21}$, $\Delta m^2_{31}$, $\delta_{CP}$, $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$.
- If we consider $\Delta m^2_{21} > 0$ then $\Delta m^2_{31}$ can be positive or negative. The two cases are named normal ordering and inverted ordering.

## Charged lepton masses

The Yukawa lagrangian for charged leptons according with $U(1)_X$ symmmetry is giving by

$$-\mathcal{L}_{Y,l} = \sum_{i,j=e.\mu.\tau} h_{e2}^{ij}\bar{\ell}_L^i \phi_2 e_R^j + \text{h.c.}$$

After symmetry braking the mass matrix is

$$\mathcal{M}_E = \begin{pmatrix} 0 & h_{e2}^{e\mu} & h_{e2}^{e\tau} \\ h_{e2}^{e\mu *} & h_{e2}^{\mu\mu} & 0 \\ h_{e2}^{e\tau *} & 0 & h_{e2}^{\tau\tau} \end{pmatrix} \frac{v_2}{\sqrt{2}}$$

where the phases can be taken out of the mass matrix as

$$\mathcal{M}_{\text{Herm},E} = \mathcal{D}^{\dagger 2,3}(\alpha,\beta)\,\mathcal{M}_E\,\mathcal{D}^{2,3}(\alpha,\beta)$$

$\mathcal{M}_E$ is a real symmetry mass matrix and $\mathcal{D}^{2,3}(\alpha,\beta)$ is the diagonal phase matrix which can be written as

$$\mathcal{M}_E = \begin{pmatrix} 0 & \eta & \zeta \\ \eta & h & 0 \\ \zeta & 0 & H \end{pmatrix} \frac{v_2}{\sqrt{2}}$$

$$\mathcal{D}^{2,3}(\alpha,\beta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

In order to have a herarquical lepton masses we will consider the following relation $\eta, \zeta \ll h \ll H$. The unitary matrix which dioganized the mass matrix is giving by

$$R_l = \begin{pmatrix} 1 & -\eta/h & -\zeta/H \\ \eta/h & 1 & -\eta\zeta/hH \\ \zeta/H & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

where the phases can be taken out by refrasing the phases of the charge leptons.

And the masses for the charged leptons are given by

$$
\begin{aligned}
m_\tau &= \frac{Hv_2}{\sqrt{2}} \\
m_\mu &= \frac{hv_2}{\sqrt{2}} \qquad\qquad\qquad . \\
m_e &= \left(\frac{\eta^2}{h} + \frac{\zeta^2}{H}\right)\frac{v_2}{\sqrt{2}}.
\end{aligned}
$$

In order to reproduce the experimental values of the masses

$$m_e = 0.511\,\mathrm{MeV}\ ,\ m_\mu = 105.66\,\mathrm{MeV}\ ,\ m_\tau = 1776.82\,\mathrm{MeV}$$

we take for the Yukawa constants the following values

$$
\begin{aligned}
H &= 3.54 \times 10^{-2} \\
h &= 2.10 \times 10^{-3} \\
\zeta &\approx 2.45 \times 10^{-4}\ ;\ \eta \approx 0
\end{aligned}
$$

## Neutrino masses

The Yukawa langrangian for neutrinos according to the $U(1)_X$ is

$$\begin{aligned} -\mathcal{L}_{Y,\nu} &= \sum_{i=e,\mu;j=e,\mu,\tau} h^{ij}_{\nu 2}\bar{\ell}^i_L\tilde{\phi}_2\nu^j_R \\ &+ \sum_{i,j=e,\mu,\tau} h^{ij}_{\chi N}\nu^{i\mathrm{T}}_R C\chi N_R + \frac{1}{2}N^{i\mathrm{T}}_R C M^{ij}_N N^j_R + \mathrm{h.c.} \end{aligned}$$

which produces a $9\times 9$ mass matrix in the base
$\mathbf{N}_L = \left(\nu_L, \nu^c_R, N^c_R\right)^{\mathrm{T}}$

$$-\mathcal{L}_{Y,\nu} = \frac{1}{2}\overline{\mathbf{N}^c_L}\mathcal{M}_\nu\mathbf{N}_L + \mathrm{h.c.}$$

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m^{\mathrm{T}}_\nu & 0 \\ m_\nu & 0 & m^{\mathrm{T}}_N \\ 0 & m_N & M_N \end{pmatrix}$$

with the following $3 \times 3$ blocks

$$m_\nu = \frac{v_2}{\sqrt{2}} \begin{pmatrix} h_{\nu 2}^{ee} & h_{\nu 2}^{e\mu} & h_{\nu 2}^{e\tau} \\ h_{\nu 2}^{\mu e} & h_{\nu 2}^{\mu\mu} & h_{\nu 2}^{\mu\tau} \\ 0 & 0 & 0 \end{pmatrix} \quad , \quad m_N = h_{\chi N}^{ij} \frac{\nu_\chi}{\sqrt{2}},$$

$$M_N = \mu_N I^{ij}$$

The matrix $\mathcal{M}_\nu$ can be diagonalize by using the inverse see saw mechanism, defining the following blocks

$$\mathcal{M}_{\nu_{6\times 3}} = \begin{pmatrix} m_{\nu_{3\times 3}} \\ 0_{3\times 3} \end{pmatrix}, \mathcal{M}_{N_{6\times 6}} = \begin{pmatrix} 0_{3\times 3} & m_{N_{3\times 3}}^{\mathrm{T}} \\ m_{N_{3\times 3}} & M_{N_{3\times 3}} \end{pmatrix}$$

Then

$$\mathcal{M}_\nu = \begin{pmatrix} 0_{3\times 3} & \mathcal{M}_{\nu_{3\times 6}}^{\mathrm{T}} \\ \mathcal{M}_{\nu_{6\times 3}} & \mathcal{M}_{N_{6\times 6}} \end{pmatrix}$$

In order to diagonalize the neutrino mass matrix $\mathcal{M}_\nu$ by blocks we introduce the *W* matrix

$$W \approx \begin{pmatrix} \left(1 - \frac{1}{2}FF^\dagger\right)_{3\times3} & F_{3\times6} \\ -F^\dagger_{6\times3} & \left(1 - \frac{1}{2}F^\dagger F\right)_{6\times6} \end{pmatrix}$$

with

$$F \approx \left(\mathcal{M}_\nu^{\mathrm{T}} \mathcal{M}_N^{-1}\right)^*$$

and two blocks $3 \times 3$ and $6 \times 6$, respectively

$$\begin{aligned} m_{\mathrm{active}3\times3} &\approx m_\nu^{\mathrm{T}} \left(m_N\right)^{-1} M_N \left(m_N^{\mathrm{T}}\right)^{-1} m_\nu \\ m_{\mathrm{heavy}6\times6} &\approx \mathcal{M}_N \end{aligned}$$

where the eigenvalues of $m_{heavy}$ is much higher than the ones of $m_{act}$.

To diagonalize $m_{heavy} = \mathcal{M}_N$ by blocks we consider the $\Omega$ matrix

$$\Omega^{\mathrm{T}}\mathcal{M}_N\Omega = \Omega^{\mathrm{T}}\begin{pmatrix} 0 & m_N \\ m_N^{\mathrm{T}} & M_N \end{pmatrix}\Omega$$

$$= \begin{pmatrix} U^* m_N^{\mathrm{diag}} U^\dagger & 0 \\ 0 & V^* M_N^{\mathrm{diag}} V^\dagger \end{pmatrix}$$

$$\Omega = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 - \frac{SS^\dagger}{2} & S \\ -S^\dagger & 1 - \frac{S^\dagger S}{2} \end{pmatrix}$$

$$S = -\frac{1}{4} m_N^{-1} M_N$$

where the masses for right handed sterile neutrinos are

$$U^* m_N^{\mathrm{diag}} U^\dagger = -m_N + \frac{M_N}{2} - \frac{1}{8} M_N m_N^{-1} M_N \approx -m_N$$

$$V^* M_N^{\mathrm{diag}} V^\dagger = m_N + \frac{M_N}{2} + \frac{1}{8} M_N m_N^{-1} M_N \approx m_N$$

However, to simplify the model we propose diagonal matrices for $m_N$ and $M_N$

$$m_N = \begin{pmatrix} h_{\chi N1} & 0 & 0 \\ 0 & h_{\chi N2} & 0 \\ 0 & 0 & h_{\chi N3} \end{pmatrix} \frac{v_\chi}{\sqrt{2}}$$

$$M_N = \mu_N \mathbb{I}_{3\times 3}$$

and the the $3 \times 3$ light neutrino mass matrix is

$$\begin{aligned} m_{\mathrm{act}} &= \begin{pmatrix} (h_{\nu 2}^{ee})^2 + (h_{\nu 2}^{\mu e})^2 \rho^2 & \times & \times \\ h_{\nu 2}^{ee} h_{\nu 2}^{e\mu} + h_{\nu 2}^{\mu e} h_{\nu 2}^{\mu\mu} \rho^2 & (h_{\nu 2}^{e\mu})^2 + (h_{\nu 2}^{\mu\mu})^2 \rho^2 & \times \\ h_{\nu 2}^{ee} h_{\nu 2}^{e\tau} + h_{\nu 2}^{\mu e} h_{\nu 2}^{\mu\tau} \rho^2 & h_{\nu 2}^{e\mu} h_{\nu 2}^{e\tau} + h_{\nu 2}^{\mu\mu} h_{\nu 2}^{\mu\tau} \rho^2 & (h_{\nu 2}^{e\tau})^2 + (h_{\nu 2}^{\mu\tau})^2 \rho \end{pmatrix} \\ &\times \frac{v_2^2}{v_\chi^2} \frac{\mu_N}{h_{\chi N1}^2} \end{aligned}$$

where $\rho = h_{\chi N1}/h_{\chi N2}$.
The light neutrino mass, $m_{\mathrm{active}}$, can be diagonalied by $U_\nu^T m_{\mathrm{active}} U_\nu$

## Solutions

- The $U_\nu$ and $\Delta m^2_{ij}$ can be written as function of $h^{ij}_{\nu 2}$ and VEV's.
- Then Pontecorvo Maki Makagawa Sakata matrix is $U_{PMNS}{=}U_l \times U_\nu$

$$\begin{pmatrix} c\theta_{12}c\theta_{13} & s\theta_{12}c\theta_{13} & s\theta_{13}e^{i\delta} \\ s\theta_{12}c\theta_{23}c\theta_{12}s\theta_{13}s\theta_{23}e^{i\delta} & c\theta_{12}c\theta_{23}s\theta_{12}s\theta_{13}s\theta_{23}e^{i\delta} & c\theta_{13}s\theta_{23} \\ s\theta_{12}s\theta_{23}c\theta_{12}s\theta_{13}c\theta_{23}e^{i\delta} & c\theta_{12}s\theta_{23}s\theta_{12}s\theta_{13}c\theta_{23}e^{i\delta} & c\theta_{13}c\theta_{23} \end{pmatrix}$$

- Defining the mixing angles by

$$\sin^2\theta_{12} = \frac{|U_{e2}|^2}{1-|U_{e3}|^2} \quad , \quad \sin^2\theta_{23} = \frac{|U_{\mu 3}|^2}{1-|U_{e3}|^2}$$
$$\sin^2\theta_{13} \;=\; |U_{e3}|^2$$

- The analysis of solar, atmospheric, reactor and accelerator neutrino oscillation experiments yields, nu-fit,

## Parameter ranges

- NuFIT 3.0 (2016)
- In the case of normal mass ordering $m1 < m2 < m3$,

$$
\begin{aligned}
\sin^2\theta_{12} &= 0.306^{+0.012}_{-0.012}\,,\ \sin^2\theta_{13} = 0.02166^{+0.00075}_{-0.00075},\\
\sin^2\theta_{23} &= 0.441^{+0.027}_{-0.021};\\
\delta &\approx 261^{+51}_{-59}\\
\Delta m^2_{21} &= 7.50^{+0.19}_{-0.17}\times 10^{-5}eV^2, \Delta m^2_{31} = 2.524^{+0.039}_{-0.040}\times 10^{-3}e
\end{aligned}
$$

- In the case of inverted mass ordering $m3 < m1 < m2$

$$
\begin{aligned}
\sin^2\theta_{12} &= 0.306^{+0.012}_{-0.012}\,,\ \sin^2\theta_{13} = 0.02179^{+0.00076}_{-0.00076},\\
\sin^2\theta_{23} &= 0.587^{+0.020}_{-0.024};\\
\delta &\approx 277^{+40}_{-46}\\
\Delta m^2_{21} &= 7.50^{+0.19}_{-0.17}\times 10^{-5}eV^2, \Delta m^2_{31} = -2.514^{+0.038}_{-0.041}\times 10^{-3}
\end{aligned}
$$

Taking the expresions for $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$, $\Delta m^2_{21}$ and $\Delta m^2_{31}$ as a function of $h^{ij}_{\nu 2}$ and with the nufit data we find the best values for the Yukawa couplings of the neutrino sector for the $U(1)_X$ model

| | Normal ordering | Inverted ordering |
|---|---|---|
| $h^{ee}_{\nu 2}$ | -0.138 $\pm$0.046 | 0.820 $\pm$0.023 |
| $h^{\mu e}_{\nu 2}$ | 0.250 $\pm$0.050 | -0.900 $\pm$0.033 |
| $h^{e\mu}_{\nu 2}$ | -0.670 $\pm$0.050 | -0.600 $\pm$0.018 |
| $h^{\mu\mu}_{\nu 2}$ | 0.260 $\pm$0.200 | -0.605 $\pm$0.048 |
| $h^{e\tau}_{\nu 2}$ | -0.610 $\pm$0.063 | 0.455 $\pm$0.038 |
| $h^{\mu\tau}_{\nu 2}$ | -0.250 $\pm$0.243 | 0.880 $\pm$0.027 |

**Table:** Set of neutrino Yukawa couplings for $v_2 = 200$ GeV, $v_\chi = 1$ TeV, $\mu_N = 500$ eV., $\rho = 1/\sqrt{2}$, $h_{\chi N_1} = 1$

Conclusion

- Find solutions to the anomaly equations without global symmetry
$$SU(2)_{q_L^a} \otimes U(3)_{U_R} \otimes U(3)_{D_R}$$
- Find solutions to the anomaly equations with big X numbers and $v_\chi \approx TeV$, but $M_{Z'} \approx g_X X v_\chi$ bigger than 14 TeV