This is the dawning of the age of Gravitational Wave Astronomy

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on behalf of the LIGO-Virgo collaboration

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SILAFAE Guatemala Nov 15 2016

Einstein equations and their linearization

Gravity is responsible for attraction (Newtonian force) and waves (radiation) much like as electromagnetism

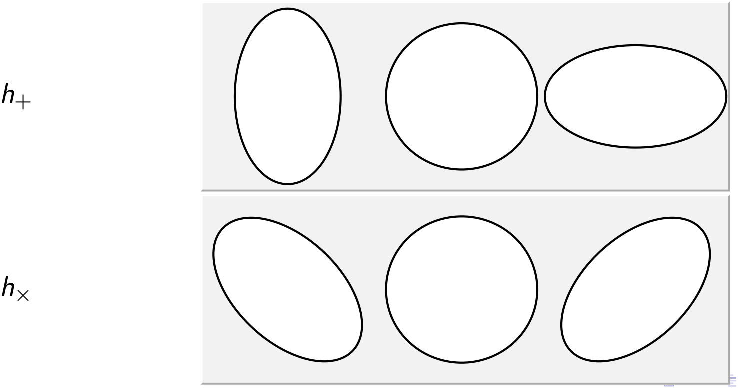
- Electromagnetism A_μ, $\mu \in (0..3)$
  1. 1 Coulomb degree of freedom, constrained by sources
  2. 2 radiative d.o.f.
  3. 1 gauge d.o.f.
- Gravity $g_{\mu\nu}$, symmetric 2-tensor, 10 components
  1. 4 constrained degrees of freedom (Newtonian potential + General Relativistic generalizations)
  2. 2 radiative d.o.f.
  3. 4 gauge d.o.f.

1&3 propagate with "the speed of thought" (Eddington '22)

GW polarizations

Gauged fixed metric after discarding $h_{0\mu}$ components, which are not radiative: transverse waves

$$h_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



The LIGO and Virgo observatories

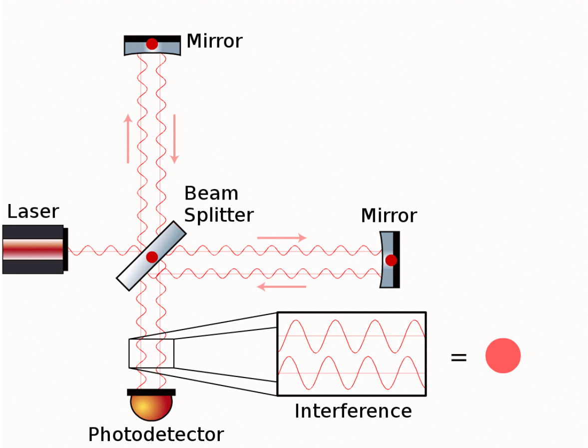
New science run due to start in 2 weeks

Last science run (O1) Sept 12th - Jan 19th.

Data ($\sim$ 130 days, with 49.6 days of actual data) analysed and published in arXiv:1606.04856

In 2020+ Japanse KAGRA and Indian INDIGO will join the collaboration

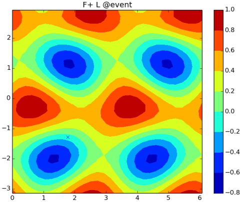
A very precise ruler

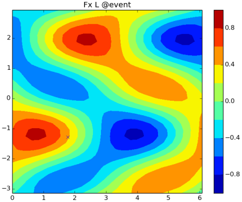


Light intensity $\propto$ light travel difference in perpendicular arms
Effective optical path increased by factor $N \sim 500$ thanks to Fabry-Perot cavities
Phase shift $\Delta\phi \sim 10^{-8}$ can be measured $\sim 2\pi N \Delta L/\lambda \to \Delta L \sim 10^{-15}/N$ m

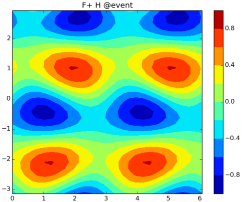
Almost omnidirectional detectors

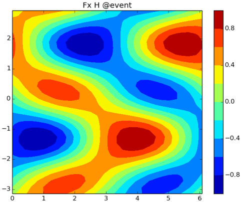
$h_{det} =$ $h_+ F_+$ $h_\times F_\times$





$F_{+,\times}$ L

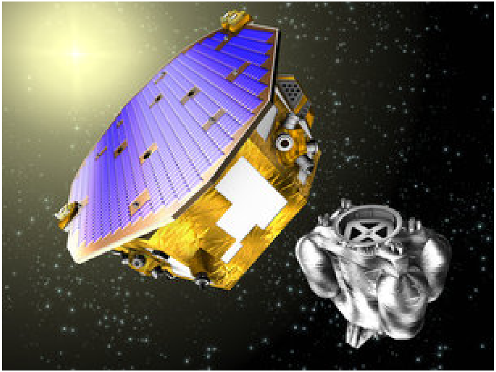




$F_{+,\times}$ H

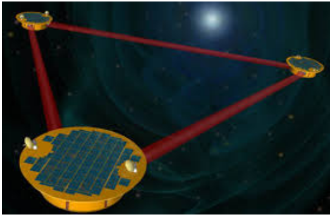
$h_{+,\times}$ depend on source, $F_{+,\times}$ on relative orientation source/detector

Space interferometers



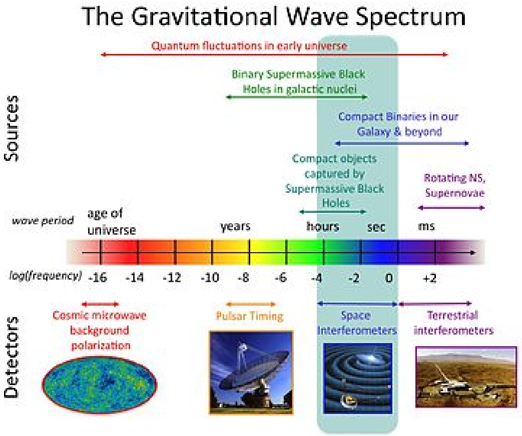
LISA pathfinder is in space, working fine!
No GW science
only technical mission

paving the way for LISA
$\sim$ 2034 (if funded)



LISA collaboration, PRL 116 (2016)

GW across the spectrum



A xilphofone of GW detectors

Wave generation: localized sources

Einstein formula relates h_{ij} to the source quadrupole moment Q_{ij}

$$\begin{aligned} Q_{ij} &= \int d^3x\rho\left(x_i x_j - \frac{1}{3}\delta_{ij}x^2\right) \qquad v^2 \simeq G_N M/r \\ h_{ij} &= \frac{2G_N}{D}\frac{d^2Q_{ij}}{dt^2} \simeq \frac{2G_N\mu v^2}{D}\cos(2\phi(t)) \end{aligned}$$

$$\begin{aligned} f &= 2\mathrm{kHz}\left(\frac{r}{30\mathrm{Km}}\right)^{-3/2}\left(\frac{M}{3M_\odot}\right)^{1/2} < f_{Max} \simeq 12kHz\left(\frac{M}{3M_\odot}\right)^{-1} \\ v &= 0.3\left(\frac{f}{1\mathrm{kHz}}\right)^{1/3}\left(\frac{M}{M_\odot}\right)^{1/3} < \frac{1}{\sqrt{6}} \end{aligned}$$

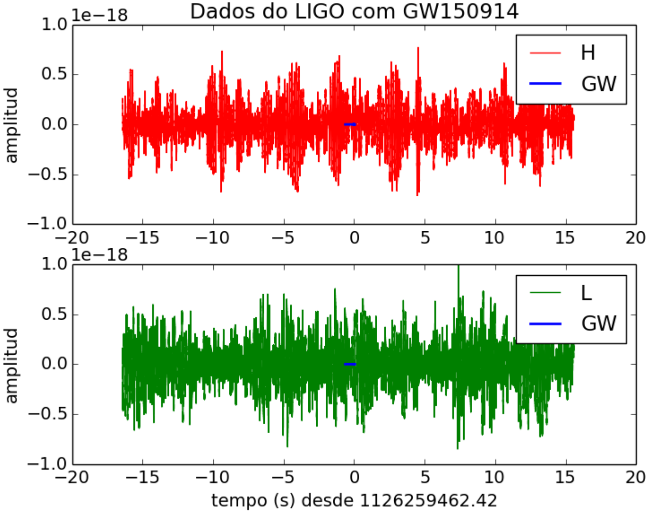
Geometric factor to keep account of transversality projection
(angular momentum L of the binary, observation direction N)

$$\begin{aligned} h_+ &\propto (1+\cos^2(\theta_{LN}))/2 \\ h_\times &\propto -\cos(\theta_{LN}) \end{aligned}$$

No direction for which emission vanishes: quadrupolar motion is bi-dimensional →
not all motion components can be collinear ($\theta_{LN} \sim \pi/2$) with N

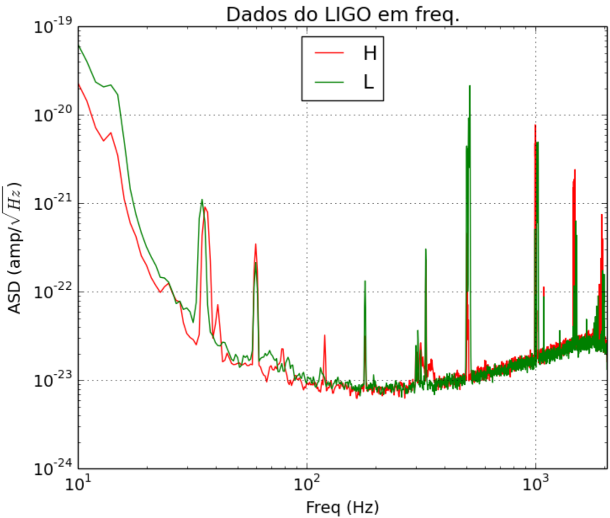
(unlike dipolar motion for the electromagnetic case)

How noisy are LIGOs?



Data from https://losc.ligo.org/events/GW150914/

Why it did not appear before?



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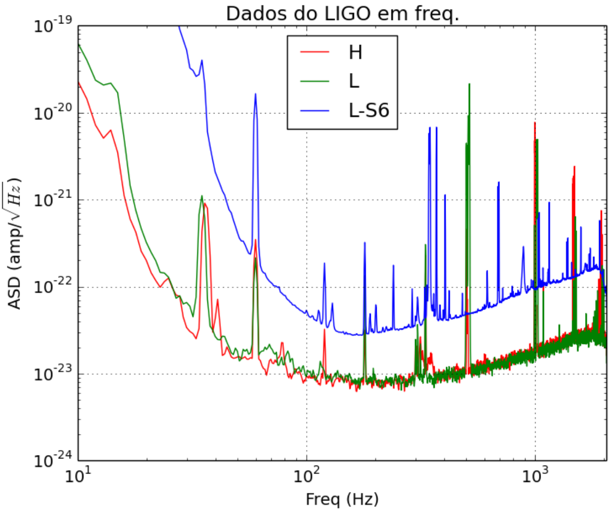

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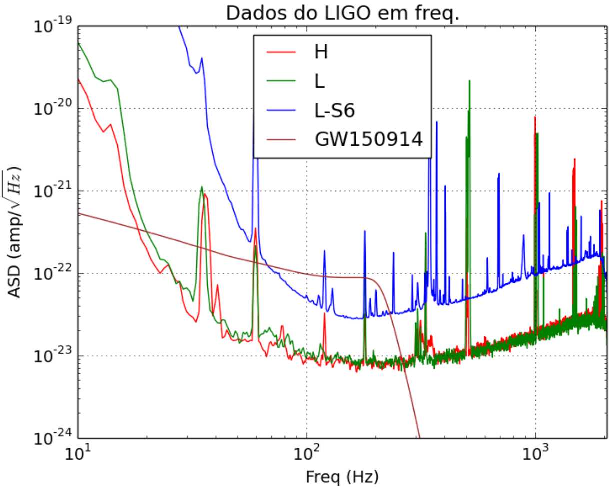

universal slope $|\tilde{h}(f)| \sim f^{-7/6}$

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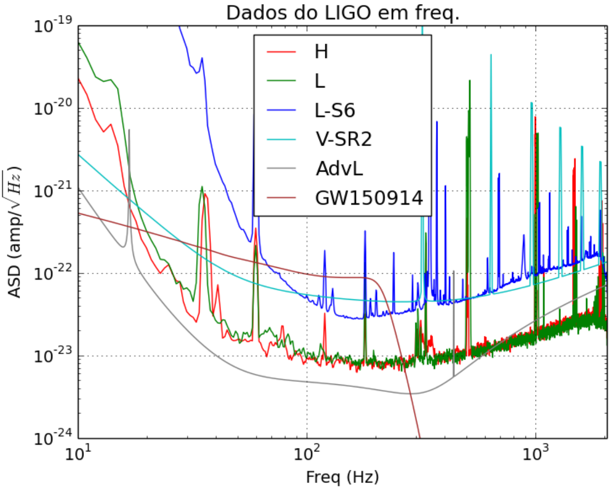
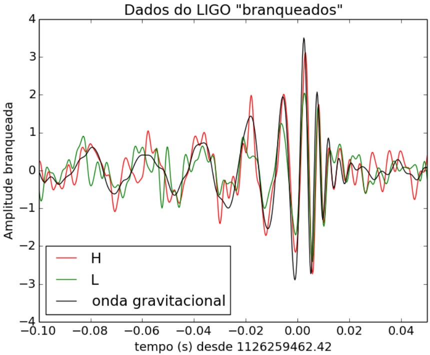


Very strong!

Data from https://losc.ligo.org/events/GW150914/

After whitening



The signal appears loud and clear:

$M_1 \sim 36 M_\odot,\ M_2 \sim 29 M_\odot$

$d_L \sim 400$ Mpc $\sim 10^{25}$ km $\sim 10^9$ light years

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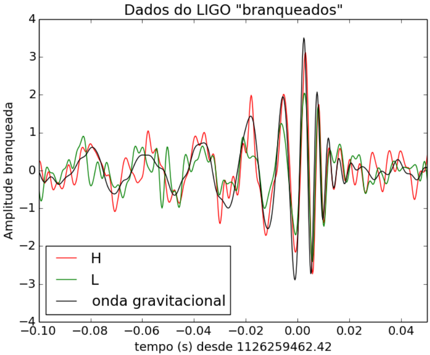

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Black holes'spins could not be measured ☹

Data from https://losc.ligo.org/events/GW150914/

Data analysis technique: Matched filtering

An experimental apparatus output: time series

$$O(t) = h(t) + n(t) \qquad h(t) = D^{ij} h_{ij}(t)$$

Noise is conveniently characterized by its spectral function

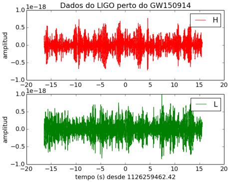
$$\langle \tilde{n}(f) \tilde{n}^*(f') \rangle = \delta(f - f') S_n(f) \qquad [Hz^{-1}]$$

Matched filter enhances the sensitivity

$$\frac{1}{T}\int_0^T O(t)h(t)\,dt = \frac{1}{T}\int_0^T h^2(t)\,dt + \frac{1}{T}\int_0^T n(t)h(t)\,dt \sim$$

$$h_0^2 + \sqrt{\frac{\tau_0}{T}} n_0 h_0$$

Best method to highlight a signal (if you know it!)



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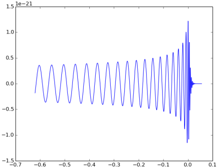

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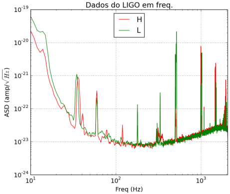

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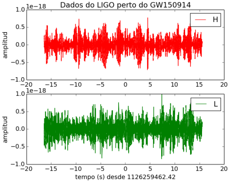

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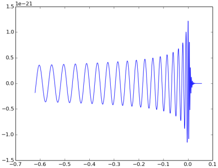

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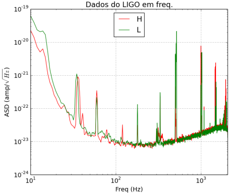

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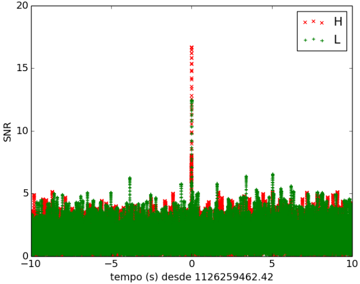
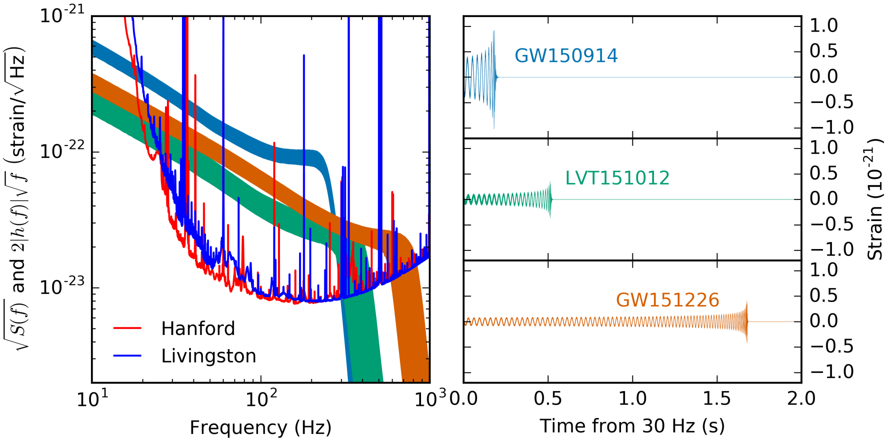


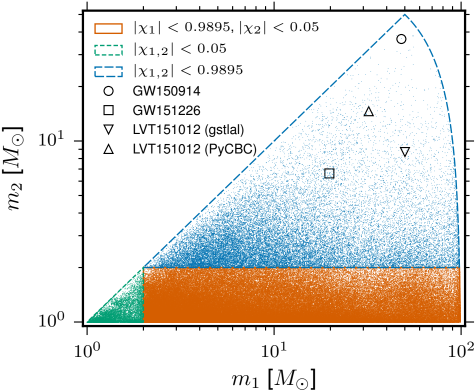
Data from https://losc.ligo.org/events/GW150914/

The 2 confirmed GW events + 1 candidate trigger



LIGO/Virgo arXiv:1606.04856

Bank



Bank with over 200k templates is prepared, matched-filtering computed against all wfs. Spin somehow neglected for low masses (anyway non-precessing) because

- astrophysically spin of neutron stars $\chi \equiv |Spin|/m^2 < 0.1$
- impractical as number of templates would explode!

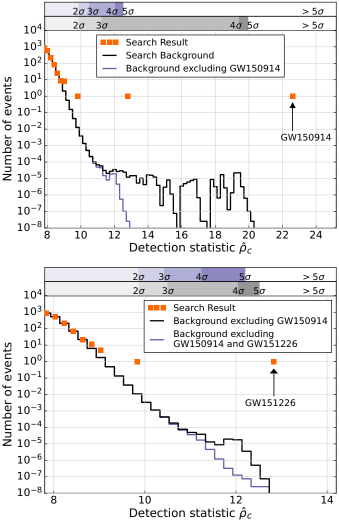
LIGO/Virgo arXiv:1606.04856

Could have it happened by chance?

What if the high number ($\sim 200k$) of trials in taking correlation with data gave a large result by chance?

The estimate of false alarms is obtained counting the number of coincidences between shifted output data between the two detectors coincidences at large delay are not GWs

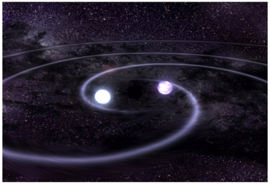
Number of coincidences at high amplitude / number of shifts $\sim 10^{-8}$



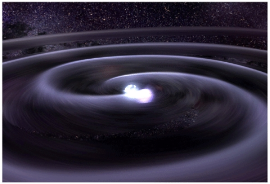
False alarm rate for GW events: $<$ 1/200.000 yrs!

LIGO/Virgo arXiv:1606.04856

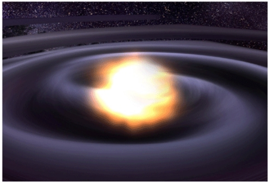
Binary coalescence: a tale made of three stories



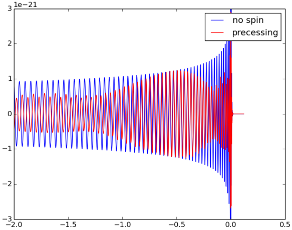
Inspiral phase post-Newtonian approximation: v/c



Merger: fully non-perturbative: Numerical Relativity



Ring-down: Perturbed Kerr Black Hole



Spin can induce precession and change the amplitude (and phase) of the waveform due to $\cos(\theta_{LN})$ factors in $h_{+,\times}$

Modeling the inspiral

Inspiral $h = A\cos(\phi(t))$ $\qquad \frac{\dot{A}}{A} \ll \dot{\phi}$

Virial relation:

$$v \equiv (G_N M \pi f_{GW})^{1/3} \qquad \nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$\begin{aligned} E(v) &= -\frac{1}{2}\nu M v^2 \left(1 + \#(\nu)v^2 + \#(\nu)v^4 + \ldots\right) \\ P(v) \equiv -\frac{dE}{dt} &= \frac{32}{5G_N} v^{10} \left(1 + \#(\nu)v^2 + \#(\nu)v^3 + \ldots\right) \end{aligned}$$

E(v)(P(v)) known up to 3(3.5)PN

$$\begin{aligned} \frac{1}{2\pi}\phi(T) &= \frac{1}{2\pi}\int^T \omega(t)dt = -\int^{v(T)} \frac{\omega(v)dE/dv}{P(v)}dv \\ &\sim \int \left(1 + \#(\nu)v^2 + \ldots + \#(\nu)v^6 + \ldots\right)\frac{dv}{v^6} \end{aligned}$$

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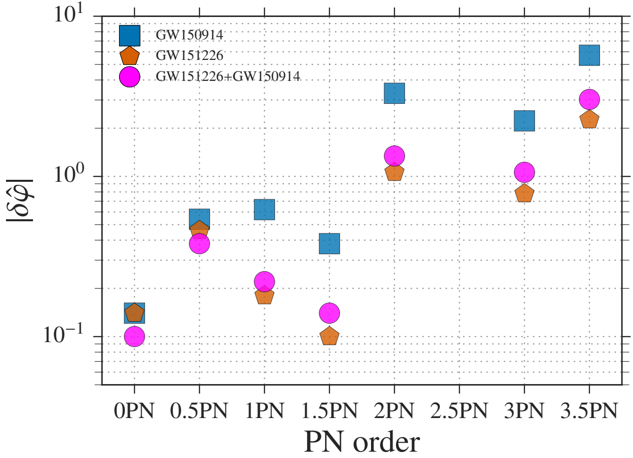
PN Coefficients

Testing the PN series with GW150914 and GW151226

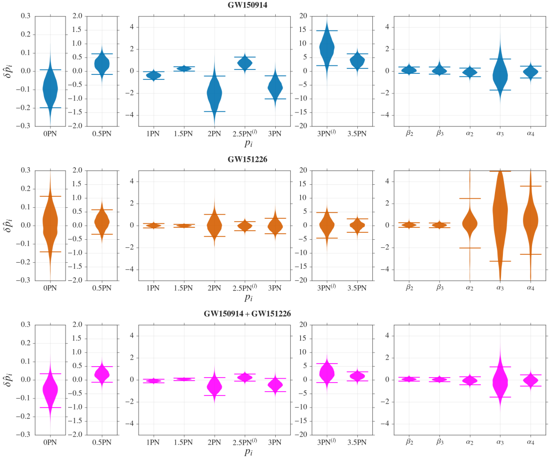


Precision in measuring the PN coefficients $\sim 100\%$: one cannot both measure the astro parameters and test GR with 1 detection! Still better precision than with binary pulsars

LIGO/Virgo arXiv:1606:04856

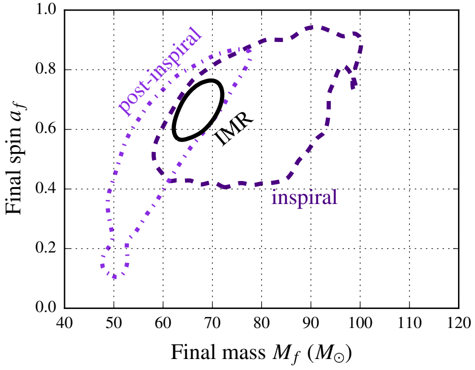
Are non-GR waveforms consistent with the 2 events?

$\phi - \phi_{GR}$: data compatible with GR



LIGO/Virgo arXiv:1606.04856

Testing GR with GW150914: comparing predictions of final Mass and Spin

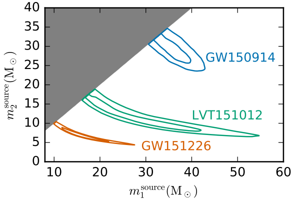


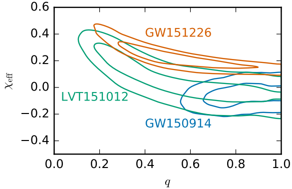
LIGO/Virgo PRL116 (216) arXiv:1602:03841

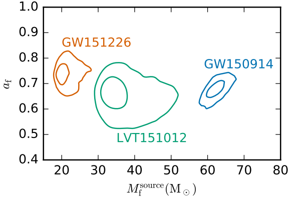
Non trivial test of GR: the final black hole mass and spin (measured from ring-down frequency and decay time) is compatible with the dynamics of two initial black holes

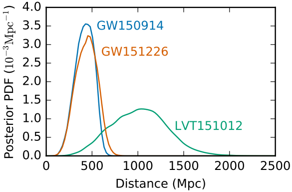
What constraints on masses/spin/distances magnitudes?

Mass determination up to 10%









$$h_+ \propto (1+\cos^2(\theta_{JN}))/2$$

$$h_\times \propto -\cos(\theta_{JN})$$

Spin magnitude $\chi_{eff}||L$

$$\chi_{eff} \equiv \left(\frac{S_1}{m_1}+\frac{S_2}{m_2}\right)\cdot\frac{L}{M}$$

LIGO/Virgo arXiv:1606.04856

How rate is estimated

In the data we have N_b background and N_f foreground triggers x_j distributed in Signal-to-Noise-Ratio x:

$$\frac{dN}{dx} = N_f p_f(x) + N_b p_b(x)$$

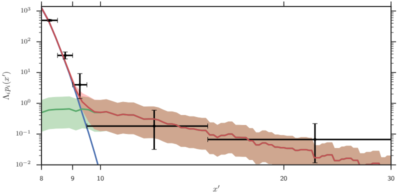
with $N_{f,b} = R_{f,b}\langle TV\rangle$, given by a Poissonian process $p(N|\mu) = \frac{\mu^N}{N!}e^{-\mu}$

$$p(N_f, N_b|x_j) \propto \mathcal{L}\left(\{x_j\}|N_f, N_b\right) p(N_f, N_b) =$$
$$\prod_{j=1}^{N=N_f+N_b}\left[N_f p_f(x_j) + N_b p_b(x_j)\right] e^{-(N_f+N_b)}$$

LIGO/Virgo arXiv:1602.03842

Splitting background and foreground

Assuming $p_f(x) \ll p_b(x)$
at $x \lesssim \bar{x} = 8 \quad \Longrightarrow$
$\int_{\bar{x}}^{x} p_f(x')dx' \sim 1 - \frac{\bar{x}^3}{x^3}$
$\langle TV \rangle$ determined empirically by injecting signals and counting how many are recovered with $x \geq \bar{x}$
$\sim$ half of them with $x \geq 10.1$
corresponding to
False Alarm Rate $< 1/$century



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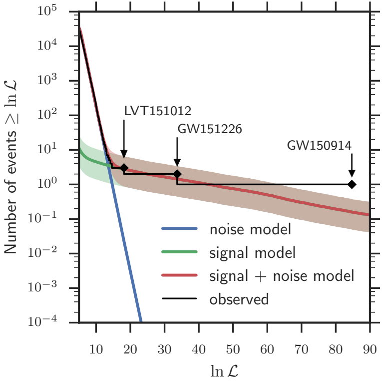
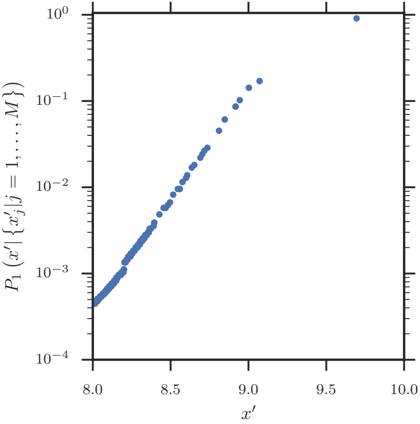


LIGO/Virgo arXiv:1606.04856

What is the inferred probability of being fore/background?

$$P_1(x) = \frac{N_f p_f(x)}{N_f p_f(x) + N_b p_b(x)}$$

Marginalizing over $N_{f,b}$ one obtains

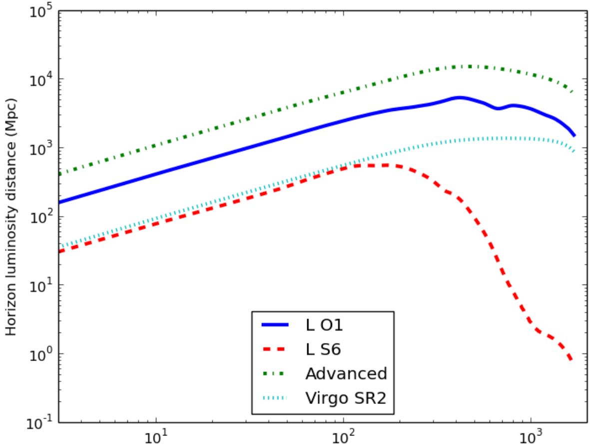


minipage

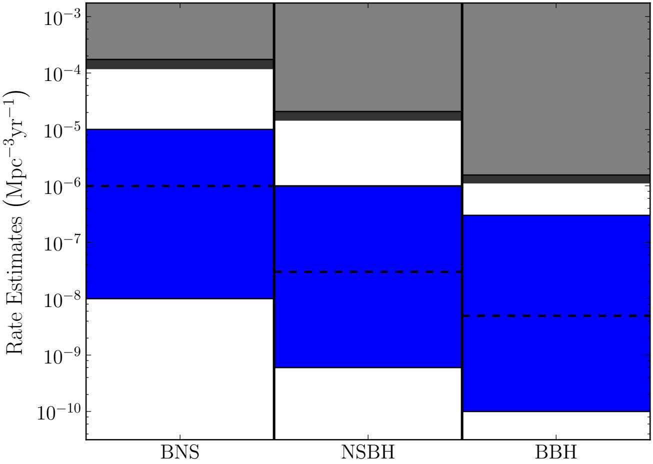
Horizon distances vs. rate estimation

$9 < \text{rate} /(Gpc^3 yr) < 240$ Galaxy density $\sim 2 \times 10^7/Gpc^3$

LIGO/Virgo PRL 116 (2016)

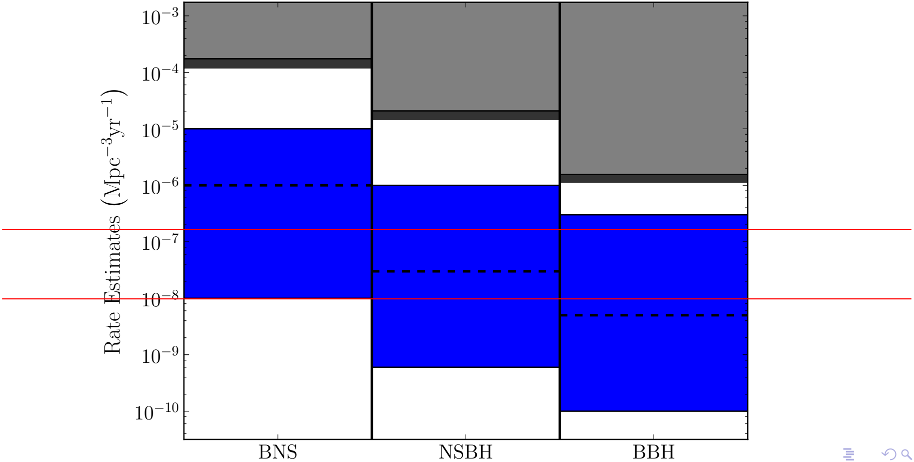


Astro predictions and previous upper bounds

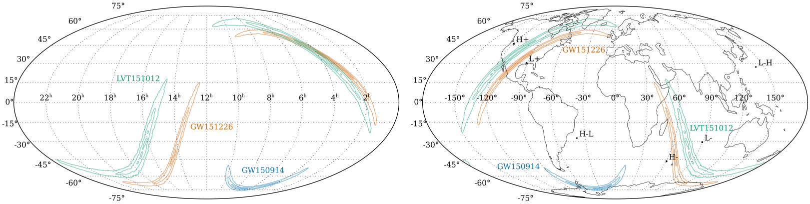


LIGO/Virgo PRD 2010

Astro predictions and previous upper bounds vs. measured BBH rate



Where do they come from?

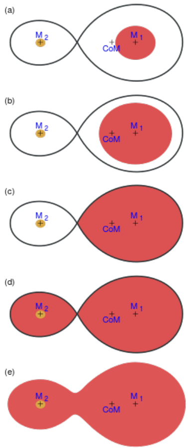


LIGO/Virgo arXiv:1606.04856

Measure of time delays (and amplitude modulation) give an estimation of the arrival direction

Triangulation will allow a better sky localization (down to ∼ few^{o2} with > 3 detectors).

How GW150914 progenitor formed?



Izzard et al. Proc. IAU 2012

A binary system of massive stars ($M_{binary} \sim 100 M_{odot}$) can go through a common envelope phase that shrinks considerably the orbit (and align the spins) → collapse to black holes

Belczynski et al. arXiv1602:04531

vs.

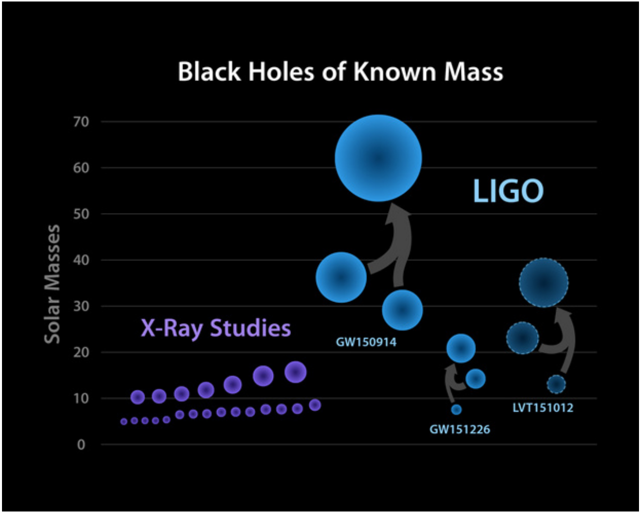
Astrophysics of GW150914

Globular Cluster: medium with high density of black holes/stars, when 3 black holes meet one is ejected and the binary shrinks

Remnants of the first stars, produced at $z \sim 6$ can give only a small contribution to the total rate

T. Hartwig et al. arXiv:1603.05655

Black holes of known mass

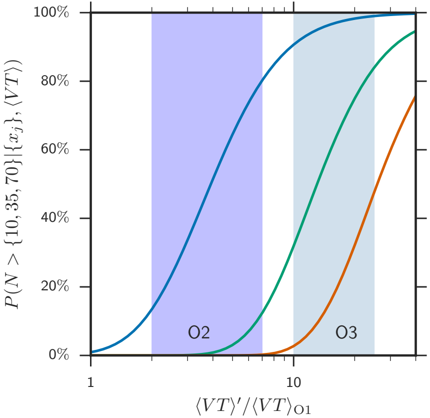


New astronomical era ahead of us!

How many detections to expect in next runs?

O2 due to start in $\sim$ December

Probability of detecting
$>$ 10, 25, 70
as a function of expected space-time sensitivity



LIGO/Virgo arXiv:1606.04856

Measuring H_0

Coalescing binary systems are standard sirens:

$$h(t) = \frac{G_N \eta M^{5/3} f_s^{2/3}}{D} \cos\left[\phi(t_s)\right]$$

In cosmological settings source and observer clocks tick differently:

$$dt_o = (1+z) dt_s \qquad f_o(1+z) = f_s$$

$$h(t_o) = \frac{G_N \eta f_o^{2/3} M^{5/3} (1+z)^{2/3}}{a(t_o) D} \qquad \cos\left[\phi(t_s(t_o))\right]$$

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$$\begin{gathered} \phi(t_s/M) = \phi(t_o/M(1+z)) \implies \\ \phi\left(t_o/\mathcal{M}\right) = \phi\left(t_s/M\right) \qquad \mathcal{M} \equiv M(1+z) \end{gathered}$$

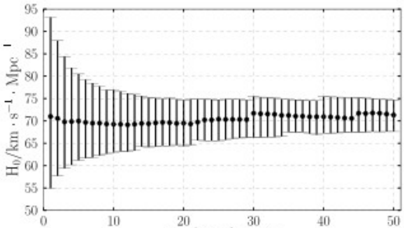
Standard sirens cosmology: Determining H_0

Hubble law: $z = H_0 d_L$

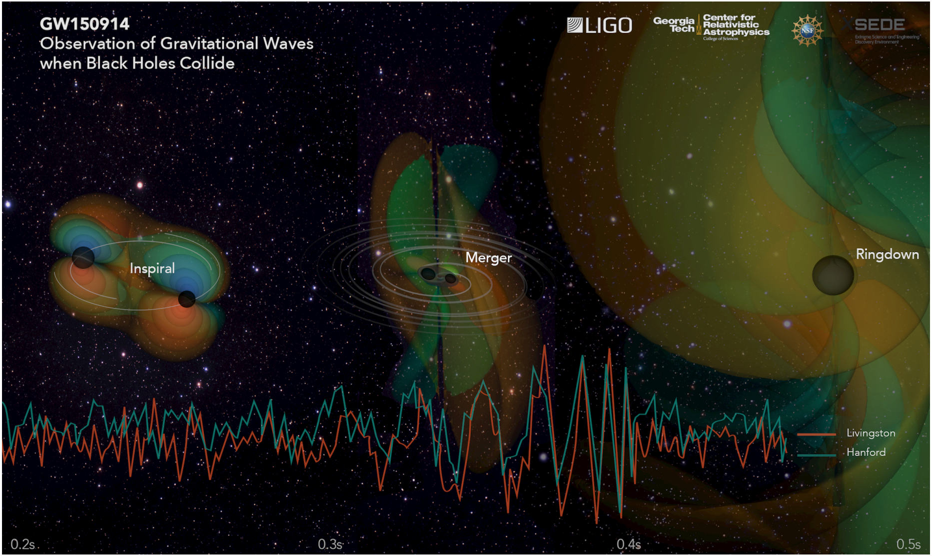
D_L can be measured, z degenerate with M, however if

- the source in the sky has been localized (α, δ)
- GW sources are in the galaxy catalog with known red-shift

$$P(z, D_L|c_i) = \int d\mathcal{M}\, d\vec{\theta}\, d\alpha\, d\delta\, P(D_L\mathcal{M}, \vec{\theta}, \alpha, \delta|c_i)\pi(z, |\alpha, \delta)$$



Schutz, Nature '86
W. Del Pozzo, arXiv:1108.1317


GW150914
Observation of Gravitational Waves
when Black Holes Collide
LIGO
Georgia Tech
Center for Relativistic Astrophysics
College of Sciences
NSF
XSEDE
Extreme Science and Engineering Discovery Environment
Inspiral
Merger
Ringdown
Livingston
Hanford
0.2s
0.3s
0.4s
0.5s

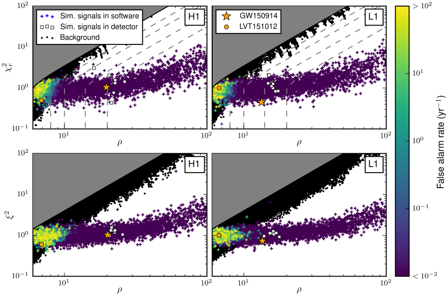
Extra slides

Noise-signal discrimination: χ^2 test

Divide signal in p frequency bands $[f_{i_1}, f_i]$: if consistent with the model the signal must match the template with almost equal power in each band (B.Allen, PRD71, 2005)

$$\chi_r^2 \propto \sum_{i=1}^{p} \left| (s|h)_{f_{i_i}<f<f_i} - (s|h)/p \right|^2$$

Re-weighted SNR:
$\hat{\rho} \equiv \frac{\rho}{[1+(\chi_r^2)^3/2]^{1/6}}$



LIGO/Virgo
PRD93 (2016),
arXiv:1602.03839

How long are signals expected to be?

Longer signals (f_{peak} higher) compete with less background:

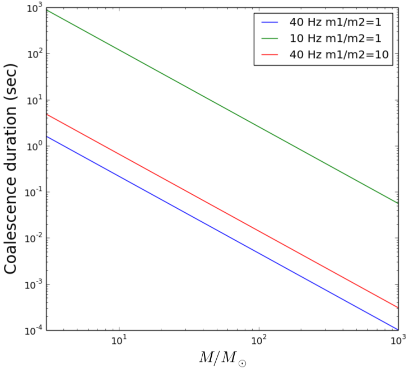
$f_{peak} \in (70, 220) Hz$

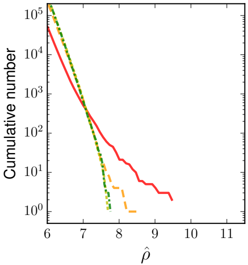
$f_{peak} \in (220, 650) Hz$

$f_{peak} \in (0.65, 2.2) kHz$

$f_{peak} \in (2, 5.9) kHz$

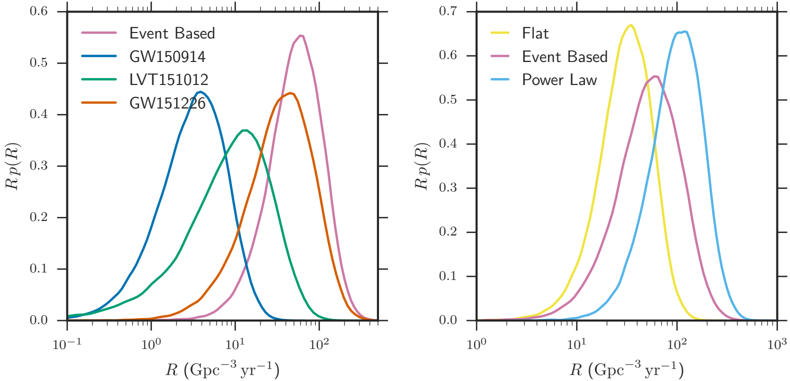
L





LIGO/Virgo PRD93 (2016), arXiv:1602.03839

Rate probability distribution functions



LIGO/Virgo arXiv:1606.04856

Was it necessary to build a detector? The Hulse-Taylor binary pulsar

GW's first observed in the NS-NS binary system PSR B1913+16
Observation of orbital parameters ($a_p \sin\iota$, e, P, $\dot\theta$, γ, $\dot P$)

determination of m_p, m_c (1PN physics, GR)

Energy dissipation in GW's → $\dot P^{(GR)}(m_p, m_c, P, e)$ vs. $\dot P^{(obs)}$

$$\frac{1}{2\pi}\phi = \int_0^T \frac{1}{P(t)}\,dt \simeq \frac{T}{P_0} - \frac{\dot P_0}{P_0^2}\frac{T^2}{2}$$

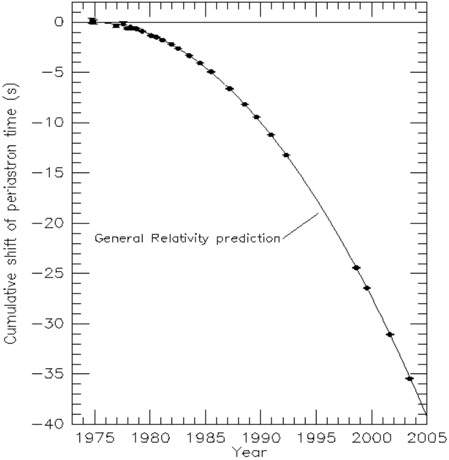
- Test of the 1PN conservative

$$E(v) = -\frac{1}{2}\nu M v^2\left(1 + \#(\nu)v^2 + \#(\nu)v^4 + \ldots\right)$$

- leading order dissipative dynamics

$$F(v) \equiv -\frac{dE}{dt} = \frac{32}{5G_N}v^{10}\left(1 + \#(\nu)v^2 + \#(\nu)v^3 + \ldots\right)$$

Weisberg and Taylor (2004)



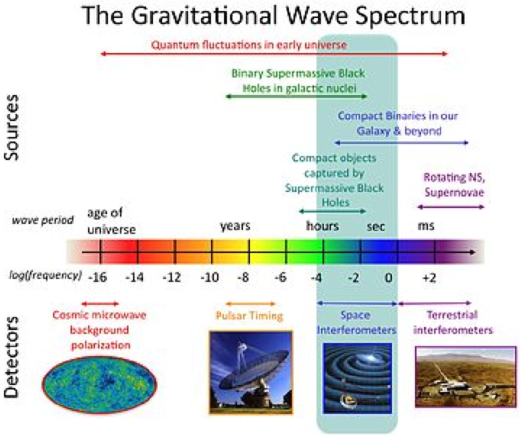
$$\frac{\dot{P}_{GR}-\dot{P}_{exp}}{\dot{P}} \sim 10^{-3}$$

10 pulsars in NS-NS, still ∼ 100Myr for coalescence

GWs and Black Holes

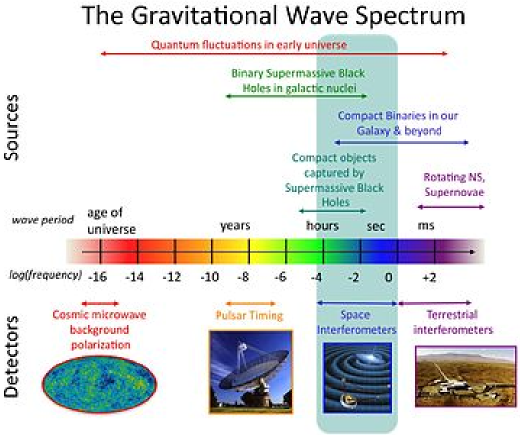
- Do black holes exist? Yes!
- If black, how to observe them?
  By light emitted by ordinary matter falling into them (until now)
- What is their mass?
  More common ones weigh 3-20 $M_\odot$, our galaxy hosts at its center a $10^6 M_\odot$ black hole.
  Galaxies can host super-massive black holes up to $10^9 M_\odot$

GW across the spectrum



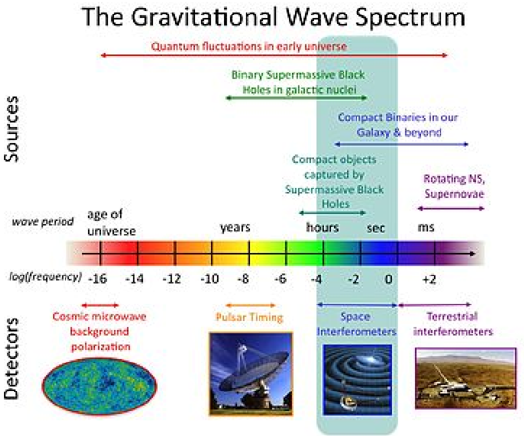
x

GW across the spectrum



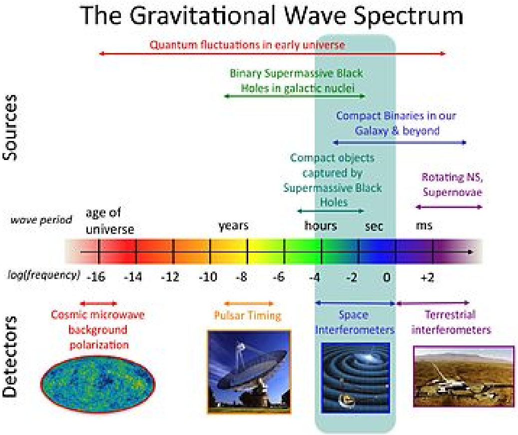
Imprint on the polarization Bx modes of CMB

GW across the spectrum



x Arrival times or radio pulses from neutron stars

GW across the spectrum



x

Interferometers: space and ground based