Improved Reissner-Nordström-(A)dS Black Hole in Asymptotic Safety

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Introduction

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- Two options for improving GR. At the level of **effective action** and improving the **solutions** (this work).

Classical RN Black Holes

Some considerations about classical RN BH

- Metric :

$$ds^2 = -f_{cl}(r)dt^2 + \frac{dr^2}{f_{cl}(r)} + r^2 d\Omega^2 \ ,$$

where

$$f_{cl}(r) = 1 - \frac{2G_0 M_0}{r} + \frac{G_0 Q_0^2}{\alpha_0 r^2} - \frac{1}{3}\Lambda_0 r^2 \ . \qquad (1)$$

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- Kretschmann scalar:

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{48G_0^2M_0^2}{r^6} + \frac{56G_0^2Q_0^4}{\alpha_0^2 r^8} - \frac{96G_0^2M_0Q_0^2}{\alpha_0 r^7} + \frac{8}{3}\Lambda_0^2 \qquad (3)$$

Classical RN Black Holes

- Possible Horizons: $f(r) = 0$

$$r_{1,2} = \rho^{1/2} \mp \left[\frac{3}{2\Lambda_0} - \rho - \frac{3G_0M_0}{2\Lambda_0}\rho^{-1/2}\right]^{1/2}, \tag{4}$$

$$r_{3,4} = - \rho^{1/2} \mp \left[\frac{3}{2\Lambda_0} - \rho + \frac{3G_0M_0}{2\Lambda_0}\rho^{-1/2}\right]^{1/2}, \tag{5}$$

where

$$\rho = \frac{1}{2\Lambda_0}\left[1 - \frac{\mathcal{R}^{-1/3}\mathcal{R}_2}{2\alpha_0} - \frac{\mathcal{R}^{1/3}}{2\alpha_0}\right], \quad \mathcal{R} = \mathcal{R}_1 + \sqrt{{\mathcal{R}_1}^2 - {\mathcal{R}_2}^3}, \tag{6}$$

$$\mathcal{R}_1 = \alpha_0^3 + 12G_0Q_0^2\alpha_0^2\Lambda_0 - 18G_0^2M_0^2\alpha_0^3\Lambda_0,$$

$$\mathcal{R}_2 = \alpha_0^2 - 4G_0Q_0^2\alpha_0\Lambda_0. \tag{7}$$

Classical RN Black Holes

- Critical Mass: $\mathcal{R}_1{}^2 - \mathcal{R}_2{}^3 \geq 0$

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$$\Rightarrow \qquad 81G_0^3\alpha_0^3\Lambda_0M_0^4 - (9G_0\alpha_0^3 + 108G_0^2Q_0^2\alpha_0^2\Lambda_0)M_0^2$$
$$+24G_0Q_0^4\alpha_0\Lambda_0 + 16G_0^2Q_0^6\Lambda_0^2 + 9Q_0^2\alpha_0^2 = 0,$$

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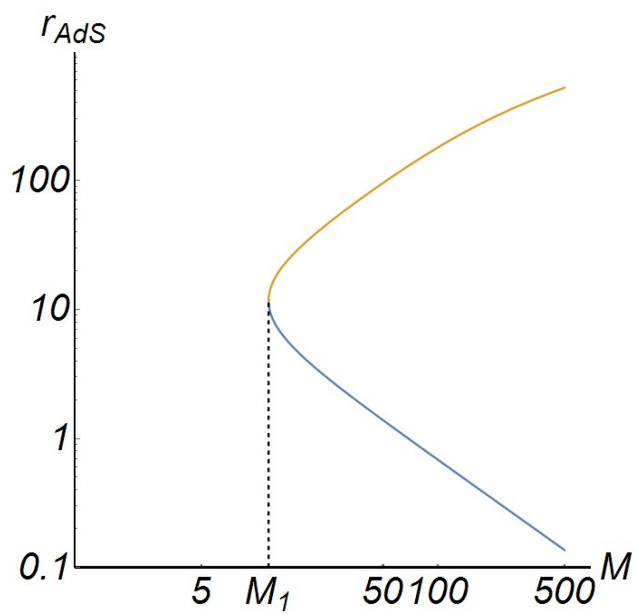
This leads to

$$M_1 = \frac{1}{3G_0}\sqrt{\frac{6G_0Q_0^2}{\alpha_0} + \frac{1}{2\Lambda_0}\left[1 - \left(1 - \frac{4G_0Q_0^2\Lambda_0}{\alpha_0}\right)^{3/2}\right]} \tag{8}$$

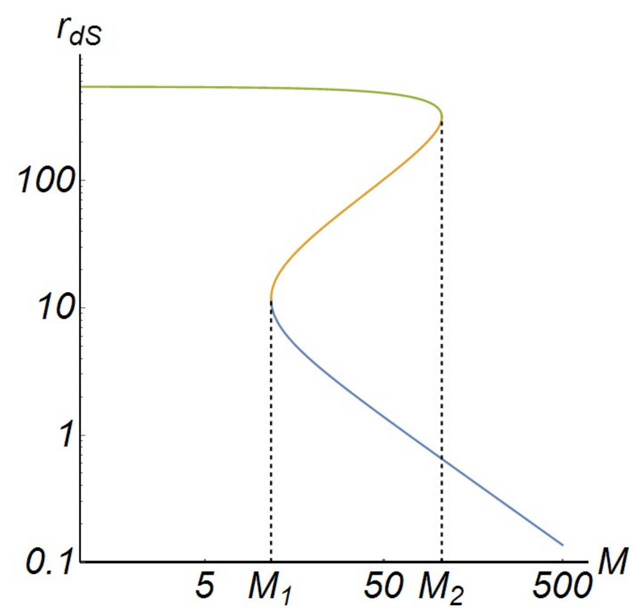
$$M_2 = \frac{1}{3G_0}\sqrt{\frac{6G_0Q_0^2}{\alpha_0} + \frac{1}{2\Lambda_0}\left[1 + \left(1 - \frac{4G_0Q_0^2\Lambda_0}{\alpha_0}\right)^{3/2}\right]} \,. \tag{9}$$

Classical RN Black Holes

- AdS BH ($\Lambda_0 < 0$): One critical mass, M_1, and two horizons, $r_{1,2}$.
- dS BH ($\Lambda_0 > 0$): Two critical masses and three horizons, $r_{1,2,4}$.



(c)



(d)

Parameters chosen $G_0 = 1, Q_0 = 1, \alpha_0 = 1/137, \Lambda_0 = \pm 10^{-5}$, then $M_1 \approx 11{,}7$, $M_2 \approx 105{,}6$.

Classical RN Black Holes

- Cosmological horizon:

$$r_c = \sqrt{\frac{3}{2\Lambda_0}} \left(1 + \sqrt{1 + \frac{4G_0 Q_0^2 \Lambda_0}{3\alpha_0}} \right)^{1/2} = r_4\big|_{M=0}$$

and in the limit $Q_0 \to 0$, one obtain

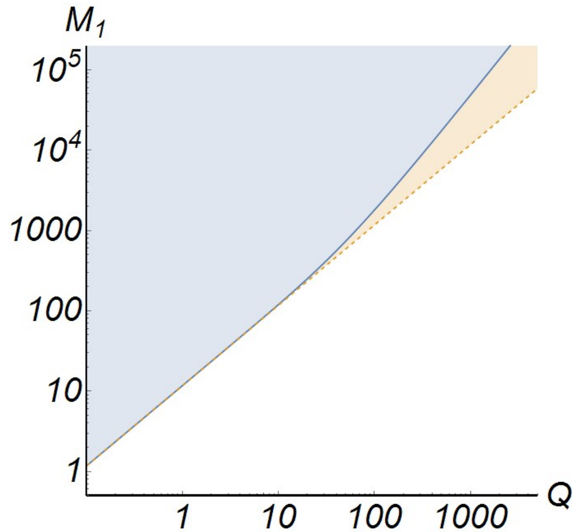
$$r_c \to \sqrt{\frac{3}{\Lambda_0}}$$

$\Rightarrow$ **dS-Schwarszchild cosmological horizon!**

Classical RN Black Holes

- Cosmic Censorship:
  Equation (8) is the condition for "dressed" singularities. In the limit $\Lambda_0 \to 0$ we have

$$M_1 \to \frac{Q_0}{\sqrt{G_0 \alpha_0}} = M_{\mathsf{critRN}}$$



In the $Q_0 \to 0$ limit, M_2 is the the Nariai BH $M_{2\mathsf{crit}} = 1/3G_0\sqrt{\Lambda_0}$.

Renormalization Group Flows in Quantum Einstein Gravity Coupled to QED

The effective action used is *Daum, Harst & Reuter, 2010. Harst & Reuter, 2011.*

$$
\begin{aligned}
\Gamma_k &= \Gamma_k^{\text{grav}} + \Gamma_k^{\text{“QED”}} \\
&= \frac{1}{16\pi G_k}\int d^4x\sqrt{g}[-R+2\Lambda_k] - \frac{1}{4\alpha_k}\int d^4x\sqrt{g}F_{\mu\nu}F^{\mu\nu} \quad , \qquad (10)
\end{aligned}
$$

where the three coupling constants are, the Newton coupling G_k, the cosmological coupling Λ_k, and the electromagnetic coupling α_k (in terms of the fine structure “constant”)

Renormalization Group Flows in Quantum Einstein Gravity Coupled to QED

The scale dependence of those couplings is indicated by the subindex k which has energy dimension one. The dimensionless couplings are obtained from the dimensionfull couplings by multiplying with the corresponding power of k

$$g_k = G_k k^2, \qquad \lambda_k = \Lambda_k k^{-2}, \qquad \alpha_k = \alpha_k \quad . \tag{11}$$

The evolution of the dimensionless couplings (11) is governed by the renormalization group equations.

$$k\partial_k g_k = \beta_g(g_k, \lambda_k) \ , \qquad k\partial_k \lambda_k = \beta_\lambda(g_k, \lambda_k) \ , \qquad k\partial_k \alpha_k = \beta_\alpha(g_k, \alpha_k) \ . \tag{12}$$

Renormalization Group Flows in Quantum Einstein Gravity Coupled to QED

The beta functions are given by

$$\beta_\lambda(g,\lambda) = (\eta_N - 2)\lambda + \frac{1}{2\pi} g \left[10\Phi^1_2(-2\lambda) - 8\Phi^1_2(0) - 5\tilde{\Phi}^1_2(0)\right], \quad (13)$$

$$\beta_g(g,\lambda) = (2+\eta_N) g \ , \quad (14)$$

$$\beta_\alpha(g,\alpha) \equiv \left(A h_2(\alpha) - \frac{6}{\pi}\Phi^1_1(0) g\right)\alpha \quad , \quad (15)$$

and the anomalous dimension of the gravitation coupling is

$$\eta_N(g,\lambda) = \frac{g B_1(\lambda)}{1 - g B_2(\lambda)} \quad ,$$

Renormalization Group Flows in Quantum Einstein Gravity Coupled to QED

The B_i are functions of the adimensional constant λ and are given by

$$\begin{aligned} B_1(\lambda) &\equiv \frac{1}{3\pi}\left[5\Phi^1_1(-2\lambda) - 18\Phi^2_2(-2\lambda) - 4\Phi^1_1(0) - 6\Phi^2_2(0)\right], \\ B_2(\lambda) &\equiv -\frac{1}{6\pi}\left[5\tilde{\Phi}^1_1(-2\lambda) - 18\tilde{\Phi}^2_2(-2\lambda)\right] \quad . \end{aligned}$$

and finally, the functions Φ_i have been calculated in the "optimised cutoff" scheme *D. Litim, 2001 & 2004*

$$\Phi^p_n(w) = \frac{1}{\Gamma(n+1)}\frac{1}{(1+w)^p}, \qquad \tilde{\Phi}^p_n(w) = \frac{1}{\Gamma(n+2)}\frac{1}{(1+w)^p} \quad . \tag{16}$$

Renormalization Group Flows in Quantum Einstein Gravity Coupled to QED

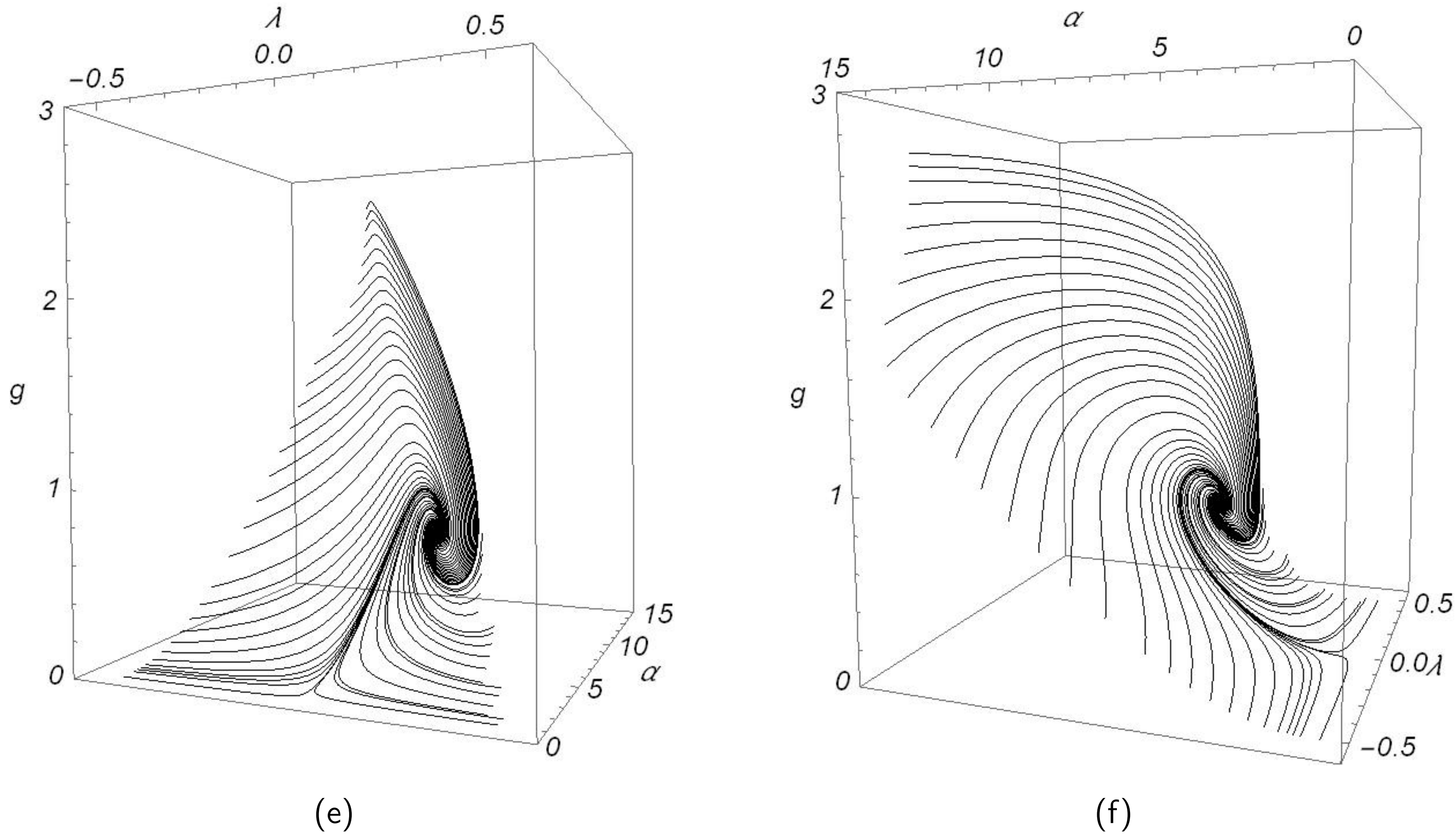
The RG equations (12) can be solved numerically. One observes, the existence of a non trivial fixed point at which $\beta_g = 0,\ \beta_\lambda = 0$ and $\beta_\alpha = 0$.

$$g_* = 0{,}707\ , \qquad \lambda_* = 0{,}193\ , \qquad \alpha_* = 6{,}365 \tag{17}$$

The dimensionfull coupling constants can be approximated at the vicinity of this fixed point

$$\lim_{k\to\infty} G_k = g_* k^{-2},\ \ \lim_{k\to\infty} \Lambda_k = \lambda_* k^2,\ \ \lim_{k\to\infty} \alpha_k = \alpha_* \quad . \tag{18}$$

Renormalization Group Flows in Quantum Einstein Gravity Coupled to QED



Renormalization Group Flows in Quantum Einstein Gravity Coupled to QED

It is sometimes convenient to work with analytic approximations of the renomalization group flow

$$g(k) = \frac{G_0 k^2}{1 + \frac{G_0}{g_*}(k^2 - k_0^2)}, \tag{19}$$

$$\lambda(k) = \frac{\Lambda_0}{k^2} + \lambda_* \left(1 - \frac{k_0^2}{k^2}\right) + \frac{g_* \lambda_*}{G_0 k^2} \log\left(\frac{1 + \frac{G_0}{g_*}k_0^2}{1 + \frac{G_0}{g_*}k^2}\right), \tag{20}$$

Renormalization Group Flows in Quantum Einstein Gravity Coupled to QED

$$\alpha(k)^{-1} = \left[1+\frac{G_0}{g_*}(k^2-k_0^2)\right]^{\frac{3\Phi g_*}{\pi}} \left[\frac{1}{\alpha_0} - \frac{g_*}{\alpha_* G_0 k_0^2}\, {}_2F_1\left(1,1,1+\frac{3\Phi g_*}{\pi};1-\frac{g_*}{G_0 k_0^2}\right)\right]$$
$$+\frac{g_*}{\alpha_* G_0 k^2}\left[1+\frac{G_0}{g_*}(k^2-k_0^2)\right]\, {}_2F_1\left(1,1,1+\frac{3\Phi g_*}{\pi};\frac{G_0 k_0^2 - g_*}{G_0 k^2}\right). \quad (21)$$

those approximated functions have the advantage that they have a well defined infra-red limit $k \to k_0$

$$g(k) \to G_0 k_0^2,\ \lambda(k) \to \Lambda_0/k_0^2,\ \alpha(k) \to \alpha_0 \quad .$$

Scale Setting for Classical Backgrounds

In the renormalization program, the coupling constants in action are promoted to scale dependent quantities

$$G \to G_k,\ \Lambda \to \Lambda_k \text{ and } \alpha \to \alpha_k. \tag{22}$$

Now, the important part of this procedure is to relate the scale k with physical quantities. Our choice is to relate the energy scale with distance by

$$k \propto 1/d.$$

We can re-obtain the usual running coupling of QED

$$\alpha^{-1}(k) = -A\ln(k) + c,$$

where $c = -A\gamma\psi(3\Phi g_*/\pi)$, γ is the Euler constant, and ψ is the Digamma function

Scale Setting for Classical Backgrounds

For our system the relation of scale and distance is of the form

$$k(r, \alpha_0, Q_0, G_0, M_0, \Lambda_0) = k(r) \equiv \frac{\xi}{d(P(r), \alpha_0, Q_0, G_0, M_0, \Lambda_0)} \quad , \tag{23}$$

where the parameter ξ controls the scale dependence, and $d(r)$ it is the proper radial distance up to a point P in a radial curve $\mathcal{C}_r$

$$d(P(r)) = \int_{\mathcal{C}_r} \sqrt{|ds^2|} \quad . \tag{24}$$

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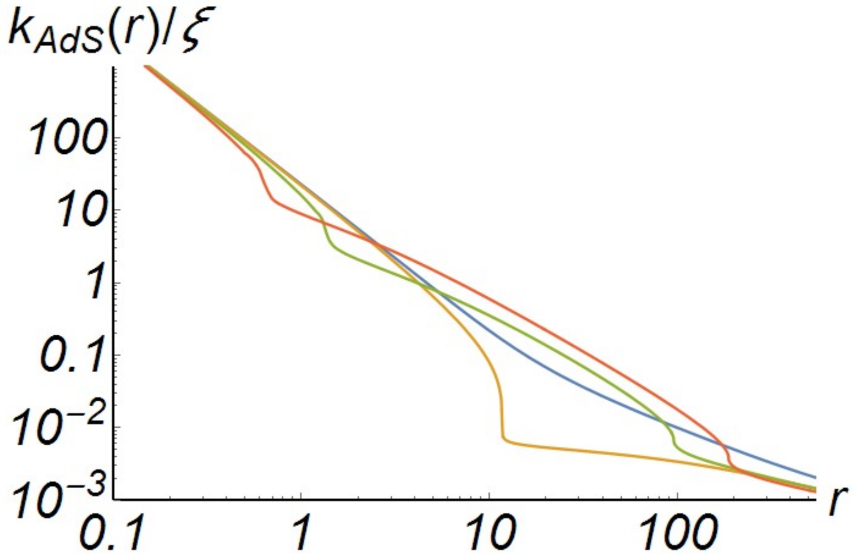
where the parameter ξ controls the scale dependence, and $d(r)$ it is the proper radial distance up to a point P in a radial curve $\mathcal{C}_r$

$$d(P(r)) = \int_{\mathcal{C}_r} \sqrt{|ds^2|} \quad . \tag{26}$$

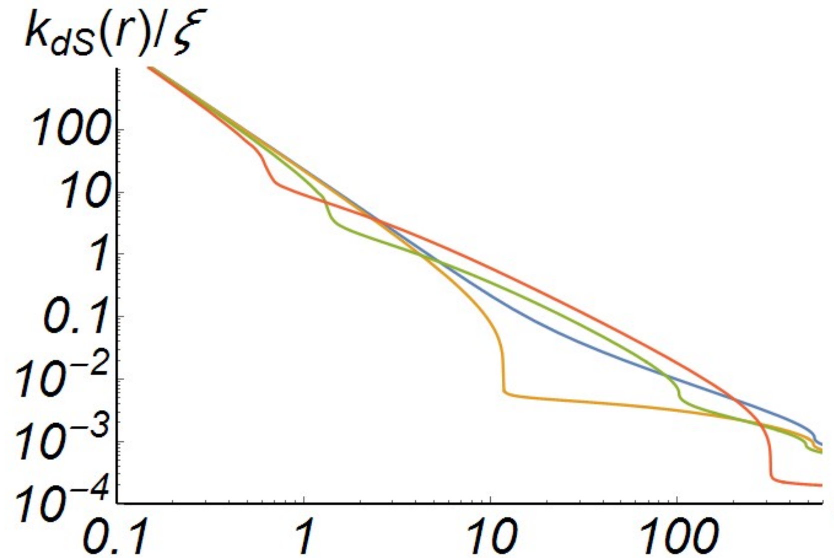
For the black hole metric (2), this length scale reads

$$d(r) = \int_0^r \frac{dr}{\sqrt{|f(r)|}} = \int_0^r \frac{dr}{\sqrt{|1 - \frac{2G_0M_0}{r} + \frac{G_0Q_0^2}{\alpha_0 r^2} - \frac{1}{3}\Lambda_0 r^2|}} \quad . \tag{27}$$

Scale Setting for Classical Backgrounds



(g)



(h)

Dependence of the scale $k(r)/\xi$ according to (25) for $G_0 = 1, Q_0 = 1, \alpha_0 = 1/137, \Lambda_0 = \pm 10^{-5}$. AdS line element with masses M_0={5, 11.7, 50, 105.6} (blue, yellow, green, and red).

Scale Setting for Classical Backgrounds

We can compute an analytical solution for the pure Reissner-Nordström proper distance, setting $\Lambda_0 = 0$ in (2),

$$
\begin{aligned}
d_{\mathsf{RN}}(r) &= r\sqrt{f_{\mathsf{RN}}(r)} + G_0 M \log\left| r\sqrt{f_{\mathsf{RN}}(r)} + r - G_0 M \right| \\
&\quad + G_0 M \log\left| Q\sqrt{\frac{G_0}{\alpha_0}}\left(1 - \frac{M}{Q_0}\sqrt{\alpha_0 G_0}\right)\right| - Q\sqrt{\frac{G_0}{\alpha_0}} \quad , \qquad (28)
\end{aligned}
$$

with $f_{\mathsf{RN}}(r) = f_{cl}(r)|_{\Lambda_0=0}$. This solution only applies for masses different from the critical mass .

RG-Improved Black Hole Solutions in the UV

An analytical expression for $k(r)$ not be obtained, unless for very extreme regimes, such as very small radial coordinates, where one might dare to make an expansion in r

$$d(r) \simeq \frac{1}{2}\sqrt{\frac{\alpha_0}{G_0 Q_0^2}} r^2 \left[1 + \frac{2}{3}\frac{M_0 \alpha_0^2}{Q_0^2} r + \mathcal{O}(r^2)\right]. \tag{29}$$

The cosmological constant Λ_0 does not appear in this expansion until order r^6. With this expression for $d(r)$, at first order in r, one can get an analytic result for $k(r)$ given by

$$k(r) \simeq 2Q_0\sqrt{\frac{G_0}{\alpha_0}} r^{-2}\xi. \tag{30}$$

RG-Improved Black Hole Solutions in the UV

Taking the UV limit, $k \to \infty$, one must use (18) in

$$f_k(r) = 1 - \frac{2g(k)M_0}{k^2 r} + \frac{g(k)Q_0^2}{\alpha(k)k^2 r^2} - \frac{1}{3}\lambda(k)k^2 r^2 \quad , \tag{31}$$

obtaining

$$f_*(r) = 1 - \frac{2g_* M_0}{k^2 r} + \frac{g_* Q_0^2}{\alpha_* k^2 r^2} - \frac{1}{3}(\lambda_* k^2) r^2. \tag{32}$$

Using the analytic expression for $k(r)$, one gets

$$f_*(r) = 1 - \frac{\alpha_0 g_* M_0}{2G_0 \xi^2 Q_0^2} r^3 + \frac{\alpha_0 g_*}{4\alpha_* G_0 \xi^2} r^2 - \frac{4G_0 \lambda_* \xi^2 Q_0^2}{3\alpha_0 r^2} \ . \tag{33}$$

RG-Improved Black Hole Solutions in the UV

With this new function, in order to get an analytic expression for ξ, one can calculate a new $d(r)$, given by

$$d_*(r) \simeq \frac{\sqrt{3}}{4}\frac{1}{Q_0\xi}\sqrt{\frac{\alpha_0}{G_0\lambda_*}}r^2. \quad (34)$$

Comparing the first order improvement (29) and the second order improvement (34), one finds that both length scales agree, indicating a (UV) convergence of the improvements if one chooses

$$\xi^2 = \xi_{sc}^2 \equiv \frac{3}{4\lambda_*} \quad . \quad (35)$$

This UV-stable choice is similar to the choice which was in previous studies called "self consistent". *Koch & Saueressig, 2014.*

RG-Improved Black Hole Solutions in the UV

and now we have the "self-consistent" improved function

$$f_{*,sc}(r) = 1 - \frac{2\alpha_0 g_* \lambda_* M_0 r^3}{3G_0 Q_0^2} + \frac{\alpha_0 g_* \lambda_* r^2}{3\alpha_* G_0} - \frac{G_0 Q_0^2}{\alpha_0 r^2} \; . \tag{36}$$

One can compute the square of the Riemman-tensor of the UV RG-Improved solution (36), obtaining

$$\begin{aligned} R^{*,sc}_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}_{*,sc} \quad = \quad & \frac{8\alpha_0^2 g_*^2 \lambda_*^2}{3\alpha_*^2 G_0^2} + \frac{304\alpha_0^2 g_*^2 \lambda_*^2 M_0^2 r^2}{9G_0^2 Q_0^4} \\ & - \frac{160\alpha_0^2 g_*^2 \lambda_*^2 M_0 r}{9\alpha_* G_0^2 Q_0^2} + \frac{64 g_* \lambda_* M_0}{3r^3} + \frac{56 G_0^2 Q_0^4}{\alpha_0^2 r^8} \; . \end{aligned} \tag{37}$$

where one can see that the spatial singularity at the origin $r = 0$ remains.

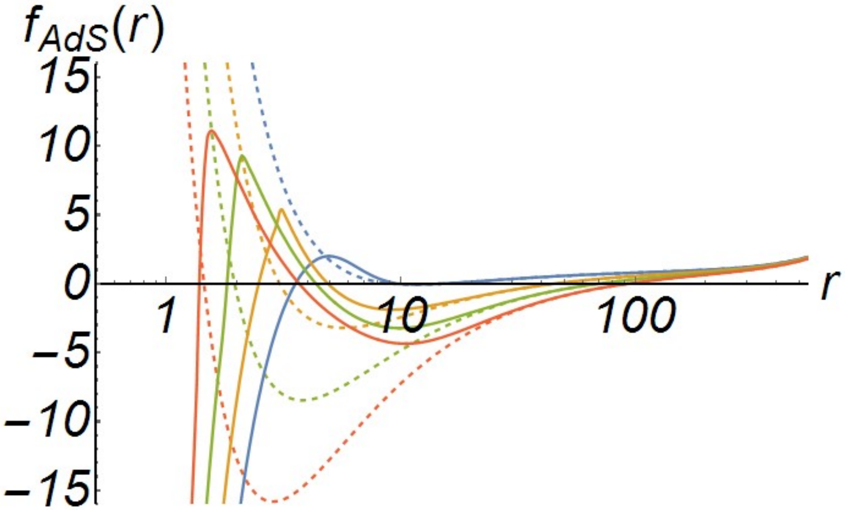
Improved Black Hole Solution in the IR and Global Behavior: Improved line element

In this improvement scheme one promotes the scale independent couplings, that are present in the classical solution, to the scale dependent quantities known from the RG flow (13)-(15)

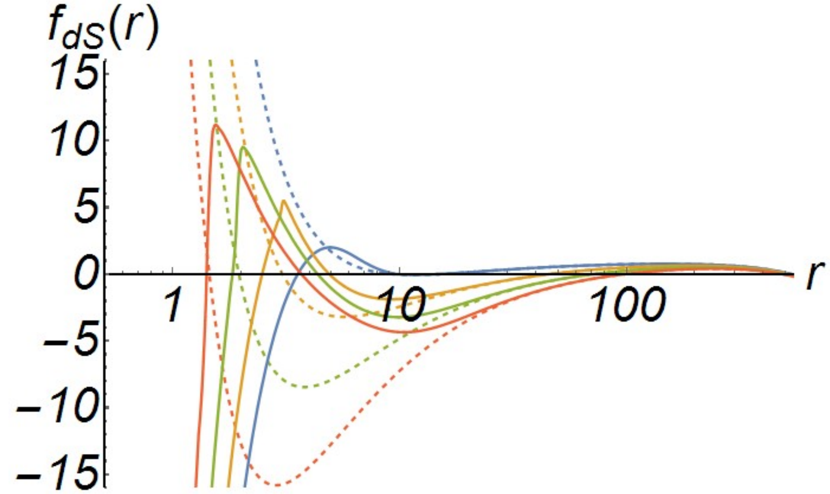
$$f_k(r) = 1 - \frac{2g(k)M_0}{k^2 r} + \frac{g(k)Q_0^2}{\alpha(k)k^2 r^2} - \frac{1}{3}\lambda(k)k^2 r^2 \quad . \tag{38}$$

The arbitrary scale k becomes a physically relevant quantity due to the scale setting (25) shown in figure . With this scale setting one obtains the RG-improved metric function $f(r)$ shown in figure .

Improved Black Hole Solution in the IR and Global Behavior: Improved line element



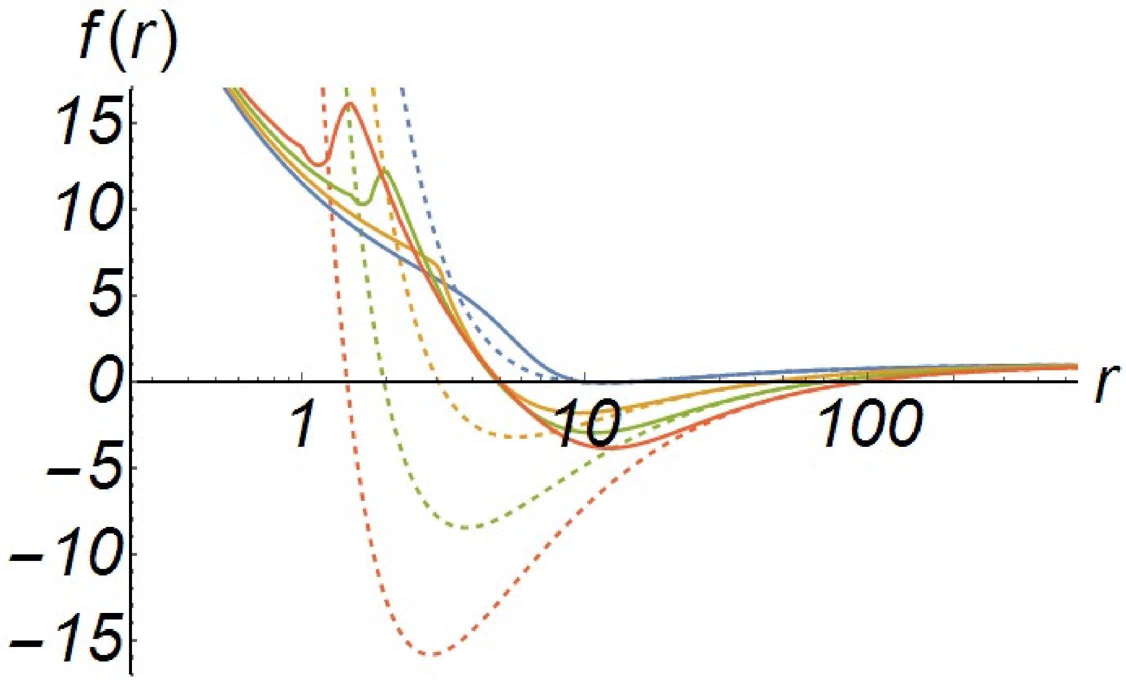
(i)



(j)

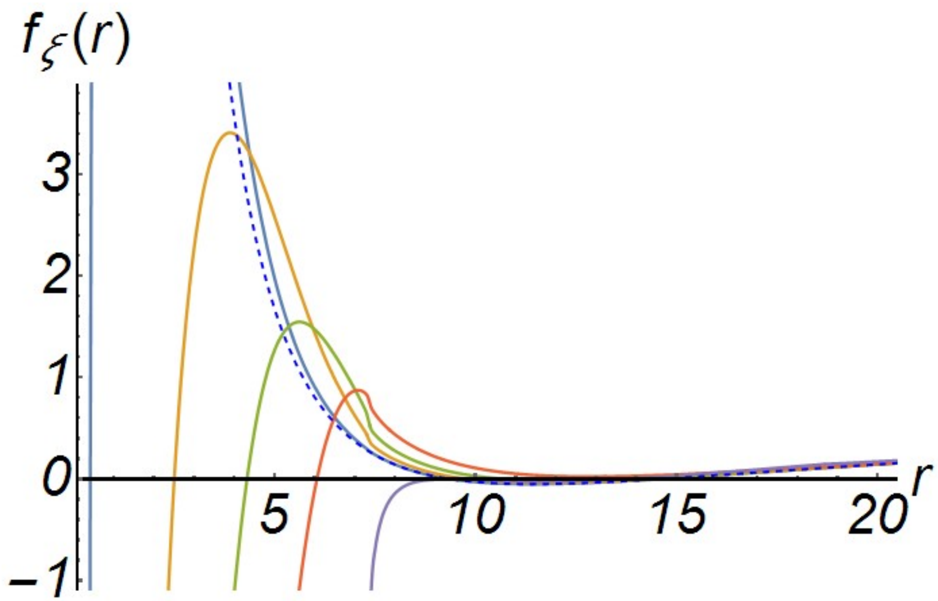
Improved metric function for the self-consistent value ξ_{sc} and with mass values M_0={12, 24, 36, 48} (blue, yellow, green, red), and $G_0 = 1, Q_0 = 1, \alpha_0 = 1/137, \Lambda_0 = \pm 10^{-5}$, $k_0 = 0{,}01$. The dashed lines are the classical metric function for each case, plotted for comparison.

Improved Black Hole Solution in the IR and Global Behavior: Improved line element



Improved metric function for the pure Reissner-Nördstrum solution, with the same parameters and color codings of previous plots

Improved Black Hole Solution in the IR and Global Behavior: Improved line element



Improved $f(r)$ for AdS case, setting the mass parameter $M_0 = 12$ and $G_0 = 1, Q_0 = 1, \alpha_0 = 1/137, k_0 = 0{,}01$. We use different ξ values, $\xi = \{0{,}5,\ 1{,}5,\ 2{,}5,\ 4,\ 11\}$ (blue, yellow, green, red, purple) and the dashed line correspond to the classical solution for comparison with the same parameters.

Alternative improvement schemes

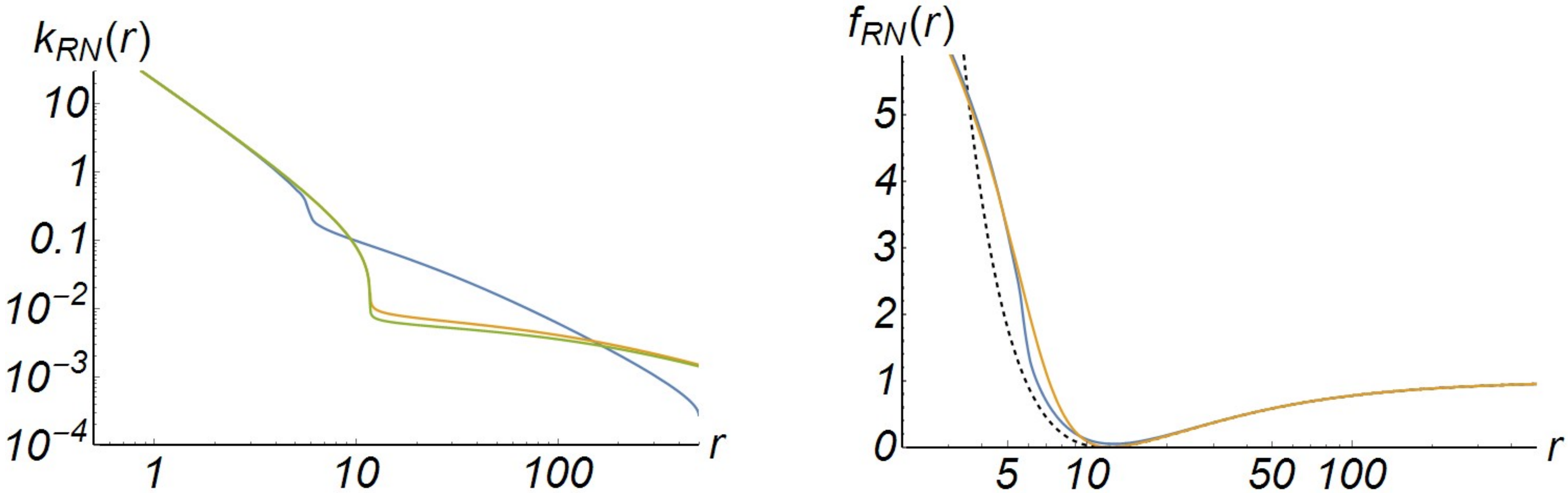
In the case of improved black hole solution an alternative choice for the renormalization scale k would be for example in terms of the proper time. The length scale associated is

$$k(r) = \frac{\xi}{\tau(r)} = \xi \left(\int_0^r dr' \left(f(r) - f(r') \right)^{-1/2} \right)^{-1} . \quad (39)$$

One can compute this scale numerically and compare it with the corresponding proper distance, numerical or analytical (28). This is done for the Reissner Nordström case

$$\tau_{RN}(r) = \int_0^r dr' \left(\frac{2G_0 M}{r'} - \frac{G_0 Q^2}{\alpha_0 r'^2} - \frac{2G_0 M}{r} + \frac{G_0 Q^2}{\alpha_0 r^2} \right)^{-1/2} . \quad (40)$$

Alternative improvement schemes



(k) (l)

Left: Comparison of the proper time (blue) and numerical distance (orange) and analytical distance (green) scale settings, using $G_0 = 1, Q_0 = 1, \alpha_0 = 1/137$ and a mass value $M \simeq M_{crit}|_{\Lambda_0=0}$.

Right: $f(r)$ improved functions for the two scales settings. We use the same values for the constants and ξ_{sc}.

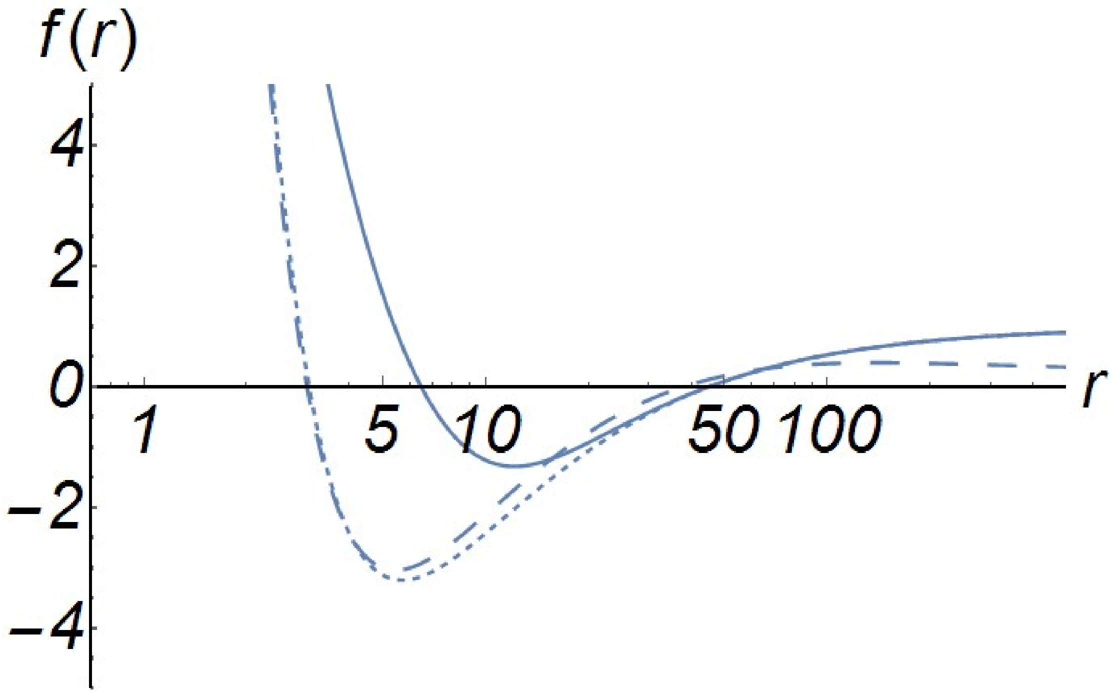
Alternative improvement schemes

It is interesting to compare the behavior of the metric functions obtained from the improving solutions approach $f_{impRN}(r)$ and the metric function that solves exactly the simplified version of the gap equations $\tilde{f}(r)$. *Koch & Rioseco, 2015.*
One finds that both quantum improved descriptions have a well defined classical limit

$$\lim_{\xi\to 0} f_{impRN}(r) = f_{RN}(r), \tag{41}$$
$$\lim_{\epsilon\to 0} \tilde{f}(r) = f_{RN}(r).$$

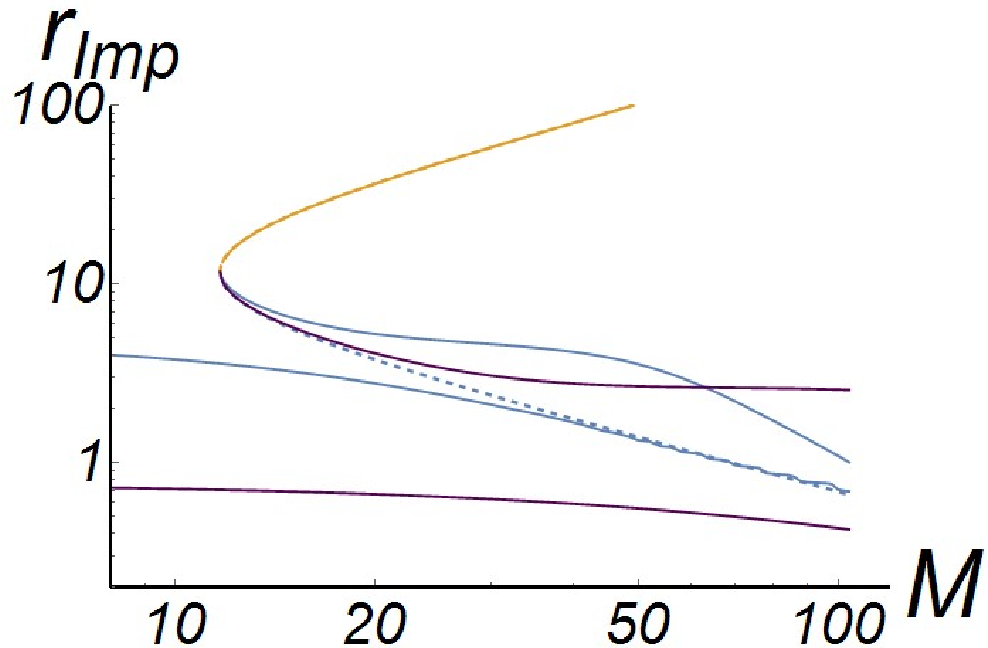
A comparison of both functions is shown in next figure for

$$\tilde{f}(r) = \frac{r^4\epsilon^2\alpha_0 + 4\epsilon r^3\alpha_0 + 4(1 - G_0M_0\epsilon)r^2\alpha_0 - 8rG_0M_0\alpha_0 + 4G_0Q_0^2}{4r^2(\epsilon r + 1)^2\alpha_0} \tag{42}$$



Comparison of $\tilde{f}(r)$ (dashed), $f_{impRN}(r)$ (line) and $f_{RN}(r)$ (point) for the Reissner-Nordstöm black hole, using ξ_{sc}, $\epsilon = 0{,}01$, $G_0 = 1, Q_0 = 1, \alpha_0 = 1/137, k_0 = 0{,}01$ and $M = 15$

Modified horizon structure and cosmic censorship

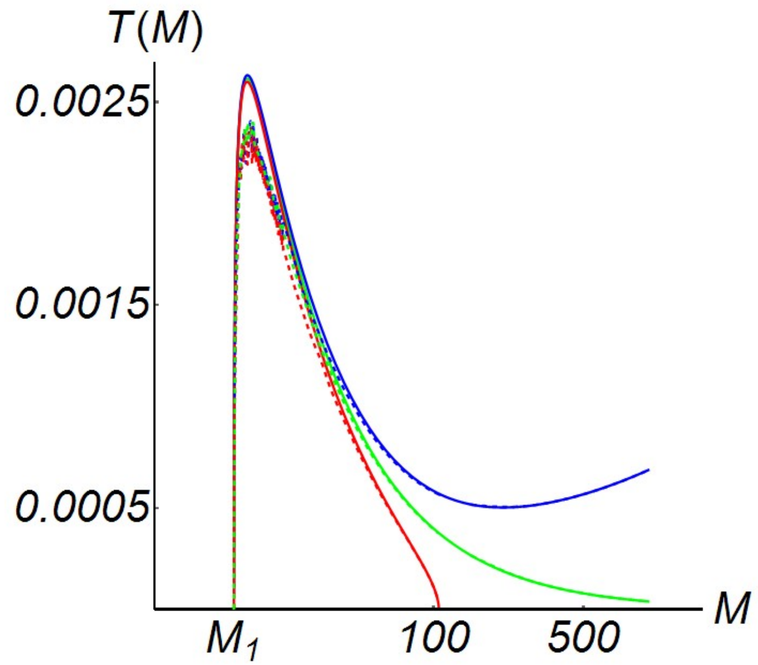


Improved dS Horizons. Orange and blue lines corresponds to external and internal horizons, respectively. Also we have purple lines, corresponding to internal horizons for $\xi = \xi_{sc}/3$. The dotted line is for the classical horizon. The values used are $G_0 = 1, Q_0 = 1, \alpha_0 = 1/137, k_0 = 0{,}01$, and ξ_{sc}. For AdS black holes we have the same behavior.

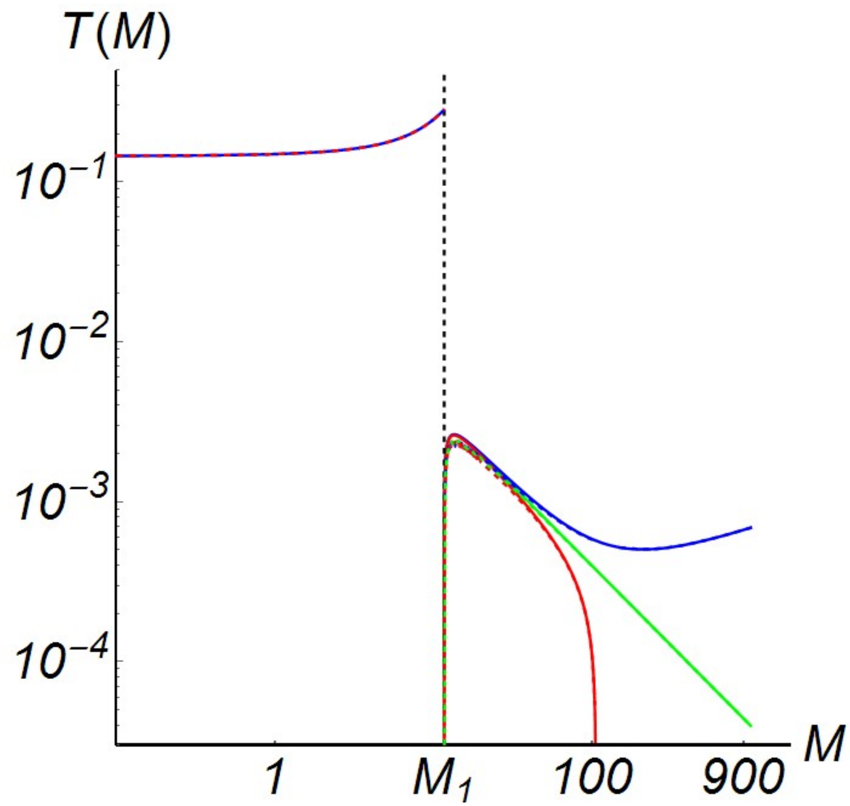
Classical and improved temperature

For describing the evaporation process for the classical BH one uses the relation

$$T = \frac{1}{4\pi} f'(r)|_{r=r_2} \quad . \tag{43}$$



Classical and improved temperature



Improved temperature as function of the mass parameter, evaluated for AdS (blue), dS (red) and $\Lambda_0 = 0$ (green) cases. Also, in full lines we have the classical temperature in r_2. The values chosen are $G_0 = 1, Q_0 = 1, \alpha_0 = 1/137, k_0 = 0{,}01$.

Modified mass and charge

It is instructive to study how one would interpret the improved black hole solution is one would not be aware of a possible scale dependence of the couplings.

In this case one would perform experiments at some radial scale r and assuming constant couplings G_0, Λ_0, and α_0.

The result of such an experiment (say the study of sections of geodesics) would then be fitted by the “charges” of the black hole.

For astrophysical distances, those charges would be basically the mass $M = M(r)$ and the electrical charge $Q^2 = Q^2(r)$, whereas the cosmological term with its corresponding “charge” $L = L(r)$ is largely irrelevant at a range of smaller radii.

Modified mass and charge

Taking equation (38) and redefining terms in sense of fitting the metric function

$$f_k(r) = 1 - \frac{2G_0 M(r)}{r} + \frac{G_0 Q^2(r)}{\alpha_0 r^2} - \frac{1}{3}\Lambda_0 L(r) r^2 \quad , \tag{44}$$

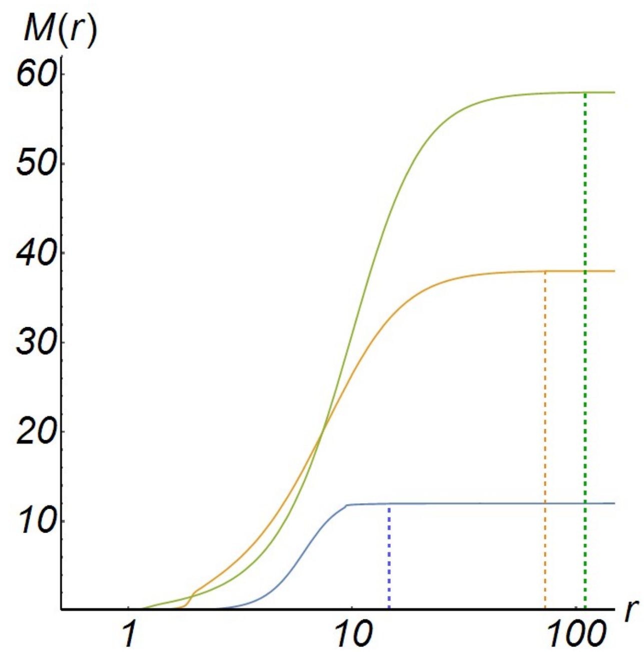
with

$$M(r) \equiv \frac{M_0 g(k)}{G_0 k^2(r)} \quad , \tag{45}$$

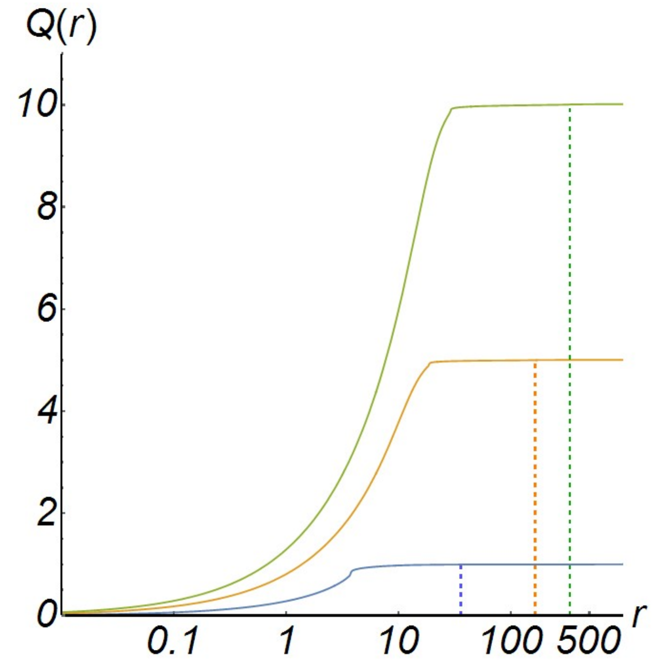
and

$$Q(r) \equiv \frac{Q_0^2 \alpha_0}{G_0} \frac{g(k)}{\alpha(k) k^2(r)} \quad . \tag{46}$$

Modified mass and charge



(m)



(n)

Mass and charge as variables dependent of r for AdS case. Left: The curves are for mass values $M_0 = \{12,\ 38,\ 58\}$ and fixed charge $Q_0 = 1$. Right: Different charge values $Q_0 = \{1,\ 5,\ 10\}$ with mass values $M_0 = \{20,\ 100,\ 250\}$, respectively. The other parameters $\xi_{sc},\ G_0 = 1,\ \alpha_0 = 1/137,\ k_0 = 0{,}01$. The dS case is basically identical since differences only would occur at extremely large radii.

Summary

- We studied some classical aspects of the charged BH solution with $\Lambda \neq 0$.
- Effects of the FRGE are applied and we analyzed how that solution changed.
- The scale dependence modified structural properties of the classical solution such as horizons.
- We found a zero order transition in temperature for the (A)dS charged BH at the critical mass M_1.
- It is important to note that the improved solution agrees with Penrose's cosmic censorship hypothesis, for any election of parameters.