Quark and Higgs sector in $7+1$ dimensional extended spin space

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Outline

SM status

- Many free parameters.
- No experimental input.
- No evidence of popular BSM.
- BSM alternatives?

Spin space extension

- Spin 1/2 fundamental degrees of freedom.
- Extra dimensions (Kaluza-Klein).
- Keep 4-d spacetime.

Clifford algebra

$\mathscr{C}_N$

- $\gamma_\alpha\gamma_\beta + \gamma_\beta\gamma_\alpha = 2g_{\alpha\beta}$.
- $\alpha, \beta = 0, 1, \ldots 3, 5, \ldots, N$.

$\mathscr{C}_4$ for spacetime.

- $\sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu, \gamma_\nu\right] \quad \text{con} \quad \mu, \nu = 0, \ldots, 3$.
- $S(\Lambda) = e^{-\frac{i}{4}\sigma_{\mu\nu}\omega^{\mu\nu}}$.

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Lorentz and scalar generators

Example

$S(\Lambda)\gamma^\mu S(\Lambda)^{-1} = \left(\Lambda^{-1}\right)^{\mu}{}_{\nu}\gamma^\nu$ para $\mu,\nu = 0,\ldots,3.$

Example

$S(\Lambda)\gamma^a S(\Lambda)^{-1} = \gamma^a$ para $a = 5,\ldots,N$

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Scalar generators

- Continuous symmetries
- Coleman-Mandula.

Scalar generators set

- $\mathscr{S}_{n-4} = \frac{1}{2}(I + \tilde{\gamma}_5)\mathrm{U}\left(2^{(N-4)/2}\right) \oplus \frac{1}{2}(I - \tilde{\gamma}_5)\mathrm{U}\left(2^{(N-4)/2}\right)$
- $\tilde{\gamma}_5 \equiv -i\gamma_0\gamma_1\gamma_2\gamma_3$

Matrix space schematics

$1-\mathscr{P}$		
	$\mathscr{S}_{(N-4)R}\otimes\mathscr{C}_4$	
		$\mathscr{S}_{(N-4)L}\otimes\mathscr{C}_4$

$1-\mathscr{P}$	$\bar{\mathrm{F}}$	$\bar{\mathrm{F}}$
F	V	S,A
F	S,A	V

General properties

Physical fields in $\mathscr{C}_4 \otimes \mathscr{S}_{N-4}$

- (elements of 3+1 space) $\times \left(\begin{array}{c} \text{combination of products of} \\ \mathscr{S}_{N-4}\text{elements} \end{array} \right)$

Transformations

- $\Psi \rightarrow U \Psi U^{\dagger}$

Eigenvalue equation

- $[\mathscr{O}, \Psi] = \lambda \Psi$

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Projection operator $\mathscr{P}$

- Acts on Poincaré $\mathscr{J}_{\mu\nu} = i\left(x_\mu \partial_\nu - x_\nu \partial_\mu\right) + \frac{1}{2}\sigma_{\mu\nu}$ and scalars $\mathscr{S}_{N-4}$ generators
$$\mathscr{J}'_{\mu\nu} = \mathscr{P}\,\mathscr{J}_{\mu\nu} = \mathscr{P}\left[i\left(x_\mu \partial_\nu - x_\nu \partial_\mu\right) + \frac{1}{2}\sigma_{\mu\nu}\right],$$
- $$\mathscr{S}' = \mathscr{P}\mathscr{S}_{N-4}.$$
- Fermions contain $1 - \mathscr{P}$ and posses standard transformation properties
- $\Psi \to U\Psi$

Electroweak Multiplets	States Ψ	I_3	Y	Q	L
Fermion doublet	$\nu_L^1 = \frac{1}{8}(1-\tilde{\gamma}_5)(\gamma^0+\gamma^3)(\gamma^5 - i\gamma^6)$	$1/2$	-1	0	1
	$e_L^1 = \frac{1}{8}(1-\tilde{\gamma}_5)(\gamma^0+\gamma^3)(1+i\gamma^5\gamma^6)$	$-1/2$	-1	-1	1
Fermion singlet	$e_R^1 = \frac{1}{8}(1+\tilde{\gamma}_5)\gamma^0(\gamma^0+\gamma^3)(\gamma^5 - i\gamma^6)$	0	-2	-1	1
Scalar doublets	$\frac{1}{4\sqrt{2}}(1-\tilde{\gamma}_5)\gamma^0(1-i\gamma^5\gamma^6)$	$1/2$	1	1	0
	$\frac{1}{4\sqrt{2}}(1-\tilde{\gamma}_5)\gamma^0(\gamma^5+i\gamma^6)$	$-1/2$	1	0	0
Vector singlets	$\frac{1}{2\sqrt{2}}\gamma^0(\gamma^1+i\gamma^2)Y$	0	0	0	0
	$\frac{1}{2}\gamma^0\gamma^3 Y$	0	0	0	0
	$\frac{1}{2\sqrt{2}}\gamma^0(\gamma^1-i\gamma^2)Y$	0	0	0	0
Vector triplet	$\frac{1}{8}(1-\tilde{\gamma}_5)\gamma^0(\gamma^1+i\gamma^2)(\gamma^5-i\gamma^6)$	1	0	1	0
	$\frac{1}{4\sqrt{2}}(1-\tilde{\gamma}_5)\gamma^0(\gamma^1+i\gamma^2)\gamma^5\gamma^6$	0	0	0	0
	$\frac{1}{8}(1-\tilde{\gamma}_5)\gamma^0(\gamma^1+i\gamma^2)(\gamma^5+i\gamma^6)$	-1	0	-1	0

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Fermion - vector Lagrangian

$$\mathscr{L} = \frac{1}{N_f} \mathrm{tr} \Psi^\dagger \{ [i\partial_\mu I_{den} + g A^a_\mu(x) I_a] \gamma_0 \gamma^\mu - M \gamma_0 \} \Psi P_f ,$$

Scalars and basis

gamma matrices

- $\gamma_0, \gamma_1, \ldots, \gamma_8$

Scalars

- 4 γ_a, $a = 5, \ldots, 8$, 6 $\gamma_{ab} \equiv \gamma_a \gamma_b$, $a < b$, 4 $\gamma_{abc} \equiv \gamma_a \gamma_b \gamma_c$, $\gamma_5 \gamma_6 \gamma_7 \gamma_8$
- 32 scalars $\mathscr{S} = P_+ \mathrm{U}(4) \oplus P_- \mathrm{U}(4)$

Cartan basis

- $1, \quad \hat{\gamma}_5, \quad \gamma_5\gamma_6, \quad \gamma_7\gamma_8, \quad \gamma_5\gamma_6\gamma_7\gamma_8, \quad \gamma_5\gamma_6\hat{\gamma}_5, \quad \gamma_7\gamma_8\hat{\gamma}_5, \quad \gamma_5\gamma_6\gamma_7\gamma_8\hat{\gamma}_5.$

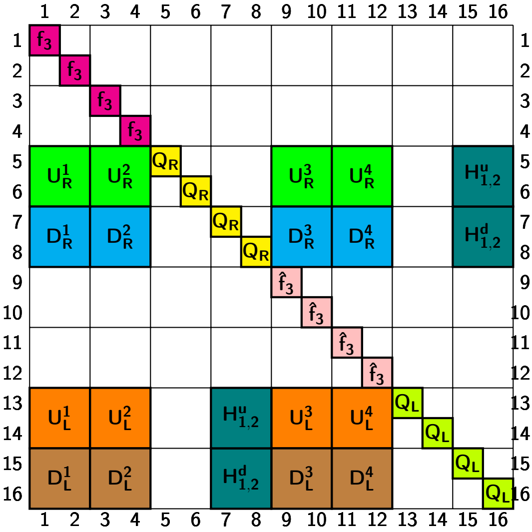
Operators

- $\mathscr{P} = B = \frac{1}{6}(1 - i\gamma_5\gamma_6).$
- $Y = \frac{1}{6}(1 - i\gamma_5\gamma_6)\left(1 + i\frac{3}{2}(1 + \tilde{\gamma}_5)\gamma_7\gamma_8\right).$
- $I_3 = \frac{i}{8}(1 - \tilde{\gamma}_5)(1 - i\gamma_5\gamma_6)\gamma_7\gamma_8.$
- $I_1 = \frac{i}{8}(1 - \tilde{\gamma}_5)(1 - i\gamma_5\gamma_6)\gamma^7.$
- $I_2 = \frac{i}{8}(1 - \tilde{\gamma}_5)(1 - i\gamma_5\gamma_6)\gamma^8$

Flavor operators

- $$f_1 = \frac{i}{8}\left(1+\tilde{\gamma}_5\right)\left(1+i\gamma^5\gamma^6\right)\gamma^7,$$
  $$f_2 = \frac{i}{8}\left(1+\tilde{\gamma}_5\right)\left(1+i\gamma^5\gamma^6\right)\gamma^8,$$
  $$f_3 = \frac{i}{8}\left(1+\tilde{\gamma}_5\right)\left(1+i\gamma^5\gamma^6\right)\gamma^7\gamma^8,$$
- $$\hat{f}_1 = \frac{i}{8}\left(1-\tilde{\gamma}_5\right)\left(1+i\gamma^5\gamma^6\right)\gamma^7,$$
  $$\hat{f}_2 = \frac{i}{8}\left(1-\tilde{\gamma}_5\right)\left(1+i\gamma^5\gamma^6\right)\gamma^8,$$
  $$\hat{f}_3 = \frac{i}{8}\left(1-\tilde{\gamma}_5\right)\left(1+i\gamma^5\gamma^6\right)\gamma^7\gamma^8,$$
- $f_0 = i\gamma^5\gamma^6\tilde{\gamma}_5, \qquad \hat{f}_0 = i\gamma^5\gamma^6.$

Matrix space schematic



Massless quarks

- $\mathbf{Q}_L^1 = \begin{pmatrix} U_L^1 \\ D_L^1 \end{pmatrix} = \begin{pmatrix} \frac{1}{16}(1-\tilde{\gamma}_5)(\gamma^5 - i\gamma^6)(\gamma^7 + i\gamma^8)(\gamma^0 - \gamma^3) \\ \frac{1}{16}(1-\tilde{\gamma}_5)(\gamma^5 - i\gamma^6)(1 - i\gamma^7\gamma^8)(\gamma^0 - \gamma^3) \end{pmatrix}$
- $U_R^1 = \frac{1}{16}(1+\tilde{\gamma}_5)(\gamma^5 - i\gamma^6)(\gamma^7 + i\gamma^8)\gamma^0(\gamma^0 - \gamma^3)$
- $D_R^1 = \frac{1}{16}(1+\tilde{\gamma}_5)(\gamma^5 - i\gamma^6)(1 - i\gamma^7\gamma^8)\gamma^0(\gamma^0 - \gamma^3)$

Baryon number 1/3, hypercharge 1/3, and polarization -1/2 (operator $\frac{3}{2}iB\gamma^1\gamma^2$), left-handed quark doublets.	I_3	Q	f_3	$\hat{f}_3$	F
$\mathbf{Q}_L^1 = \begin{pmatrix} U_L^1 \\ D_L^1 \end{pmatrix}$	$1/2$ $-1/2$	$2/3$ $-1/3$	$-1/2$ $-1/2$	0	$3/2$ $3/2$
$\mathbf{Q}_L^2 = \begin{pmatrix} U_L^2 \\ D_L^2 \end{pmatrix}$	$1/2$ $-1/2$	$2/3$ $-1/3$	$1/2$ $1/2$	0	$-1/2$ $-1/2$

Higgs doublet

Baryon number zero, Higgs-like scalars	I_3	Y	Q	$\frac{3i}{2}B\gamma^1\gamma^2$
$\phi_1 = \begin{pmatrix}\phi_1^+\\ \phi_1^0\end{pmatrix} = \begin{pmatrix}\frac{1}{8}\left(1-i\gamma^5\gamma^6\right)\left(\gamma^7+i\gamma^8\right)\gamma_0\\ \frac{1}{8}\left(1-i\gamma^5\gamma^6\right)\left(1+i\gamma^7\gamma^8\tilde{\gamma}_5\right)\gamma_0\end{pmatrix}$	$\begin{matrix}1/2\\ -1/2\end{matrix}$	1	$\begin{matrix}1\\ 0\end{matrix}$	0
$\phi_2 = \begin{pmatrix}\phi_2^+\\ \phi_2^0\end{pmatrix} = \begin{pmatrix}\frac{1}{8}\left(1-i\gamma^5\gamma^6\right)\left(\gamma^7+i\gamma^8\right)\tilde{\gamma}_5\gamma_0\\ \frac{i}{8}\left(1-i\gamma^5\gamma^6\right)\left(1+i\gamma^7\gamma^8\tilde{\gamma}_5\right)\gamma^7\gamma^8\gamma_0\end{pmatrix}$	$\begin{matrix}1/2\\ -1/2\end{matrix}$	1	$\begin{matrix}1\\ 0\end{matrix}$	0

Mass operators basis

- Hamiltonian mass operators $S\gamma_0$, $S \in \mathscr{S}$

$M_1 = \gamma^0$	$M_5 = i\tilde{\gamma}_5\gamma^0$
$M_2 = i\gamma^5\gamma^6\gamma^0$	$M_6 = \gamma^5\gamma^6\tilde{\gamma}_5\gamma^0$
$M_3 = i\gamma^7\gamma^8\gamma^0$	$M_7 = \gamma^7\gamma^8\tilde{\gamma}_5\gamma^0$
$M_4 = \gamma^5\gamma^6\gamma^7\gamma^8\gamma^0$	$M_8 = i\gamma^5\gamma^6\gamma^7\gamma^8\tilde{\gamma}_5\gamma^0$

- $$\phi_1^0 = \frac{1}{8}(M_1 - M_2) + \frac{i}{8}(M_7 - M_8),$$
  $$\phi_2^0 = \frac{1}{8}(M_3 + M_4) - \frac{i}{8}(M_5 + M_6).$$

- Choose M_1 to M_4,
- $\alpha_1(M_1 + M_2) + \alpha_2(M_3 - M_4)$ non-Higgs,
- Flavon?

Massive quarks

- $$\mathscr{M}_1 = a_1\left(\phi_1^0+\phi_1^{0\dagger}\right)+b_1\left(\phi_2^0+\phi_2^{0\dagger}\right),$$
  $$\mathscr{M}_2 = \tfrac{a_2}{4}\left(M_1+M_2\right)+\tfrac{b_2}{4}\left(M_3-M_4\right),$$
- $$U_M^1 = \tfrac{1}{2}\left(-U_L^1-U_R^1+U_L^3+U_R^3\right)$$
  $$D_M^1 = \frac{1}{2}\left(-D_L^1+D_R^1+D_L^3-D_R^3\right)$$
- $$U_M^1 = \tfrac{1}{2}\left(U_L^2-U_R^2-U_L^4+U_R^4\right),$$
  $$U_M^2 = \frac{1}{2}\left(D_L^2+D_R^2-D_L^4-D_R^4\right)$$

Massive quarks

Baryon number 1/3 and polarization -1/2 (operator $3iB\gamma^1\gamma^2$), massive quarks	Q	$\mathscr{M}_1$	$\mathscr{M}_2$	Ω
$U_M^1 = \frac{1}{2}\left(-U_L^1 - U_R^1 + U_L^3 + U_R^3\right)$	$2/3$	$\frac{a_1+b_1}{2}$	$\frac{a_2-b_2}{2}$	$\frac{a_1+b_1+a_2-b_2}{2}$
$U_M^2 = \frac{1}{2}\left(U_L^2 - U_R^2 - U_L^4 + U_R^4\right)$	$2/3$	$\frac{a_1+b_1}{2}$	$\frac{a_2+b_2}{2}$	$\frac{a_1+b_1+a_2+b_2}{2}$
$U_M^3 = -\frac{1}{2}\left(U_L^1 - U_R^1 + U_L^3 + U_R^3\right)$	$2/3$	$\frac{a_1+b_1}{2}$	$-\frac{a_2-b_2}{2}$	$\frac{a_1+b_1-a_2+b_2}{2}$
$U_M^4 = \frac{1}{2}\left(U_L^2 - U_R^2 + U_L^4 - U_R^4\right)$	$2/3$	$\frac{a_1+b_1}{2}$	$-\frac{a_2+b_2}{2}$	$\frac{a_1+b_1-a_2-b_2}{2}$
$D_M^1 = \frac{1}{2}\left(-D_L^1 + D_R^1 + D_L^3 - D_R^3\right)$	$-1/3$	$\frac{a_1-b_1}{2}$	$\frac{a_2-b_2}{2}$	$\frac{a_1-b_1+a_2-b_2}{2}$
$D_M^2 = \frac{1}{2}\left(D_L^2 - D_R^2 - D_L^4 + D_R^4\right)$	$-1/3$	$\frac{a_1-b_1}{2}$	$\frac{a_2+b_2}{2}$	$\frac{a_1-b_1+a_2+b_2}{2}$
$D_M^3 = \frac{1}{2}\left(-D_L^1 + D_R^1 - D_L^3 + D_R^3\right)$	$-1/3$	$\frac{a_1-b_1}{2}$	$-\frac{a_2-b_2}{2}$	$\frac{a_1-b_1-a_2+b_2}{2}$
$D_M^4 = \frac{1}{2}\left(D_L^2 + D_R^2 - D_L^4 + D_R^4\right)$	$-1/3$	$\frac{a_1-b_1}{2}$	$-\frac{a_2+b_2}{2}$	$\frac{a_1-b_1-a_2\,\text{[illegible]}\,b_2}{2}$

Vertical effect

- $\Omega \equiv \mathscr{M}_1 + \mathscr{M}_2$
- $a_2 = b_2 = 0$
- $$\frac{a_1 + b_1}{2}, \quad \text{for } U_M^i, \quad i = 1, \ldots, 4$$
  $$\frac{a_1 - b_1}{2}, \quad \text{for } D_M^i, \quad i = 1, \ldots, 4$$
- $$a_1 = m_u + m_d,$$
  $$b_1 = m_u - m_d$$

Horizontal effect

- $\Omega \equiv \mathscr{M}_1 + \mathscr{M}_2$
- $a_1 = b_1 = 0$
- $$\frac{a_2 - b_2}{2}, \quad \text{for } U_M^1, \quad D_M^1$$
  $$\frac{a_2 + b_2}{2}, \quad \text{for } U_M^2, \quad D_M^2$$

Summary

- Extended Clifford algebra
- 4-d spacetime
- Matrix space of operators and states
- 4 quark generations
- 2 Higgs doublets
- Mass hierarchy effects

- Outlook
  - Phenomenology
  - Texture matrices, mixing angles, top, bottom, Higgs relation.