SUPERSYMMETRY AND SPIN-EXTENDED MODEL

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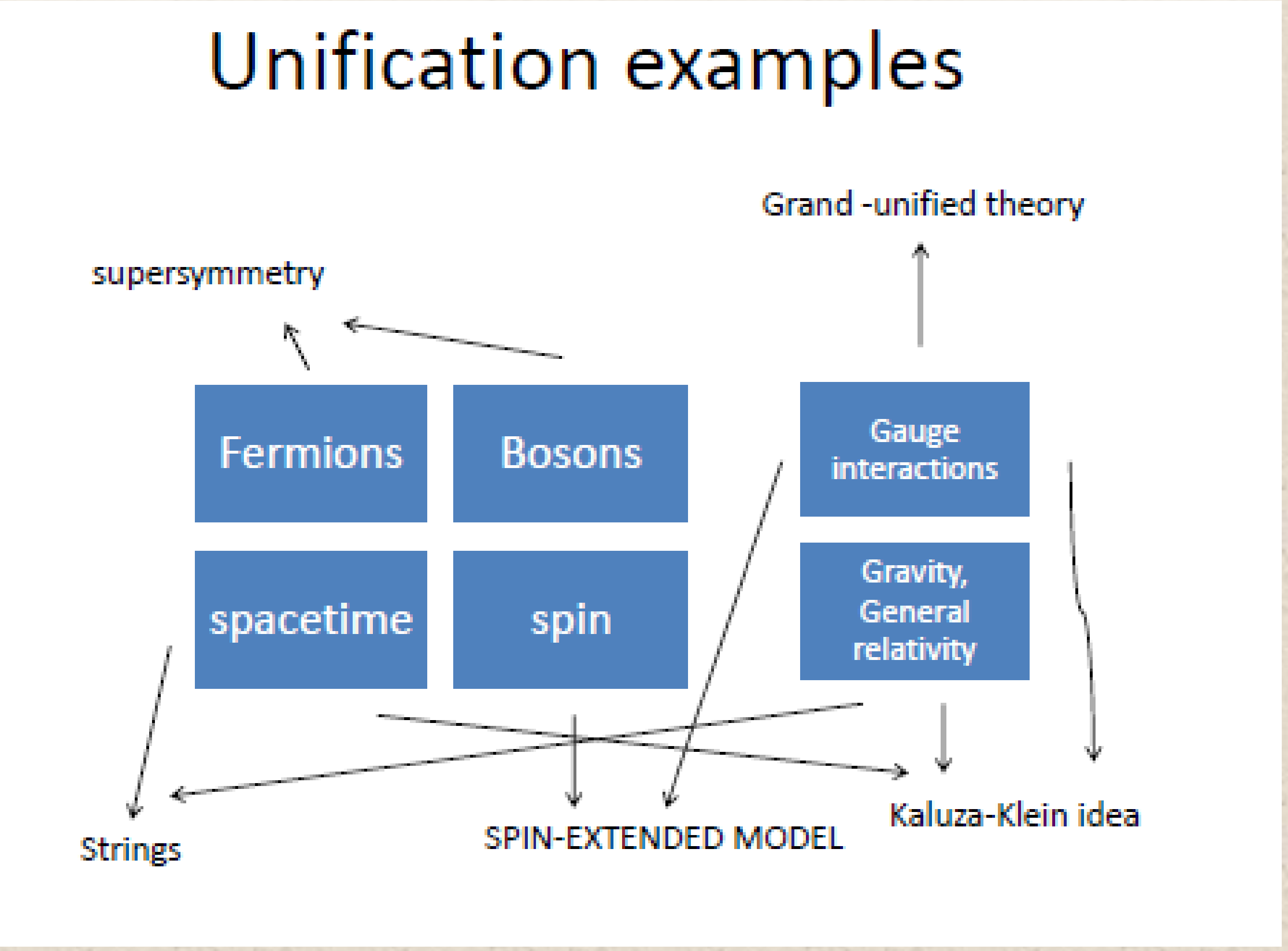
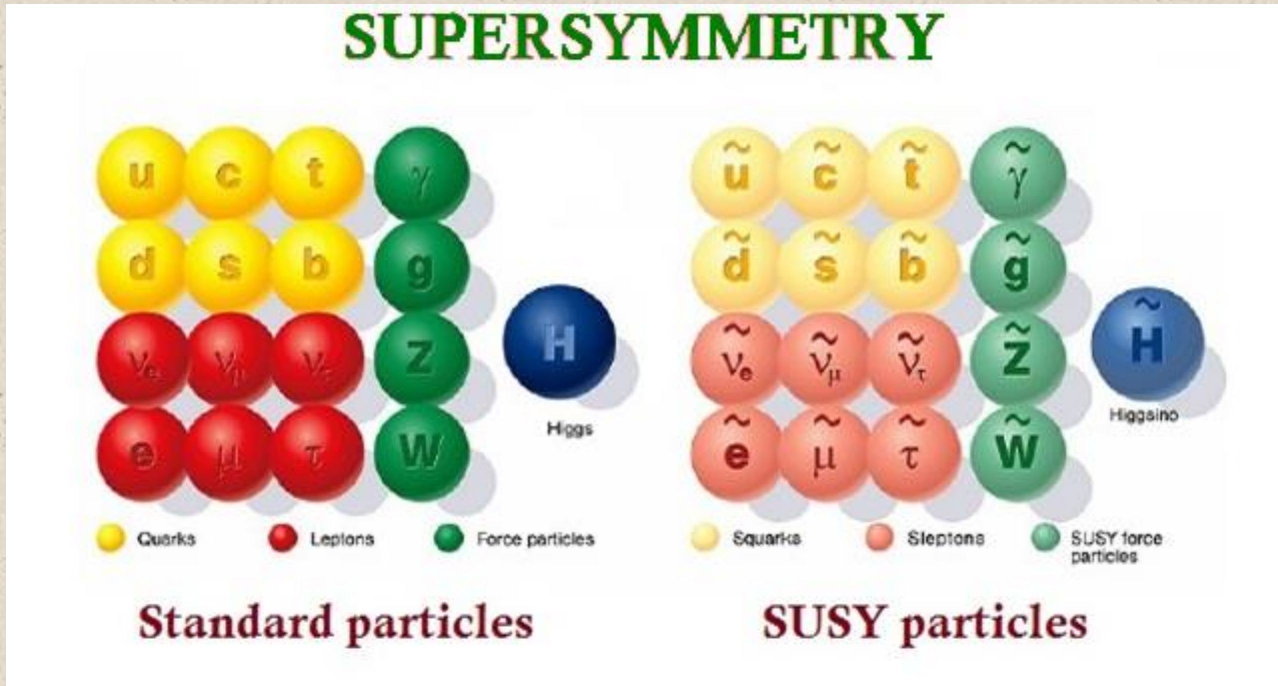
Standard-model extensions





SUSY generators must be creation and aniquilation operators that satisfy

$$Q|fermion\rangle = |boson\rangle; \qquad Q|boson\rangle = |fermion\rangle$$

$$(U_{2\pi} Q U_{2\pi}^{-1}) = -Q$$

SUSY superalgebra and Wess-Zumino model

Free Wess-Zumino Lagrangian:

$$\mathcal{L}_{WZ} = \partial_\mu \phi^\dagger \partial^\mu \phi - i\bar{\psi}_{\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \partial_\mu \psi_\alpha .$$

$$\mathcal{L}_{WZ} = \partial_\mu \phi^\dagger \partial^\mu \phi - m|\phi|^2 - i\bar{\psi}_{\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \partial_\mu \psi_\alpha + \frac{m}{2}(\psi^2 + \bar{\psi}^2).$$

⇓

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu . \ \{Q_\alpha, Q_\beta\} = 0.$$

$$[Q_a, M^{\mu\nu}] = (\sigma^{\mu\nu})_b^a Q_b$$

$$[\bar{Q}_{\dot{a}}, M^{\mu\nu}] = -\bar{Q}_{\dot{b}} (\bar{\sigma}^{\mu\nu})_a^{\dot{b}}$$

$$[Q_a, P^\mu] = [\bar{Q}^{\dot{a}}, P^\mu] = 0$$

N=1 SUPERSYMMETRY REPRESENTATIONS

Massless particles

$$\{Q_1, \bar{Q}_{\dot{1}}\} = 4E \quad , \quad \{Q_2, \bar{Q}_{\dot{2}}\} = 0$$

$$a^\dagger := \frac{\bar{Q}_{\dot{1}}}{2\sqrt{E}} \quad , \quad a := \frac{Q_1}{2\sqrt{E}}$$

$$\{a, a^\dagger\} = 1 \quad , \quad \{a^\dagger, a^\dagger\} = \{a, a\} = 0$$

$$a^\dagger|\lambda\rangle = |\lambda + \frac{1}{2}\rangle \quad , \quad a|\lambda\rangle = |\lambda - \frac{1}{2}\rangle \quad , \quad a^\dagger a^\dagger|\lambda\rangle = 0|\lambda\rangle = 0$$

Massive particles

$$\{Q_1, \bar{Q}_{\dot{1}}\} = 2E \quad , \quad \{Q_2, \bar{Q}_{\dot{2}}\} = 2E$$

$$a^\dagger_{1,2} := \frac{\bar{Q}_{\dot{1},\dot{2}}}{\sqrt{2E}} \quad , \quad a_{1,2} := \frac{Q_{1,2}}{2\sqrt{E}}$$

$$\{a_p, a^\dagger_q\} = \delta_{pq} \quad , \quad \{a^\dagger_p, a^\dagger_q\} = \{a_p, a_q\} = 0$$

$$a^\dagger_1|s_3\rangle = |s_3 + \frac{1}{2}\rangle \quad , \quad a_1|s_3\rangle = |s_3 - \frac{1}{2}\rangle \quad , \quad a^\dagger_1 a^\dagger_1|s_3\rangle = 0$$

$$a^\dagger_2|s_3 >= |s_3 - \frac{1}{2}\rangle \quad , \quad a_2|s_3\rangle = |s_3 + \frac{1}{2}\rangle \quad , \quad a^\dagger_2 a^\dagger_2|s_3\rangle = 0$$

$\lambda_0 = 0$	$\lambda_0 = -\frac{1}{2}$
$\|0\rangle$	$\|-\frac{1}{2}\rangle$
$\|\frac{1}{2}\rangle$	$\|0\rangle$

$s_0 = 0$
$\|\Omega\rangle$
$\|\frac{1}{2}\rangle$
$\|-\frac{1}{2}\rangle$
$\|\Omega'\rangle$

Spin-extended model

Introduces additional spin-like (discrete) degrees of freedom, similarly to the Kaluza-Klein idea.

For given dimension, it reproduces standard-model elements, as it predicts global or local scalar symmetries, and it also constrains the field representations.

LORENTZ AND MAXIMAL SCALAR SYMMETRY AT D DIMENSION

$\underbrace{\gamma_0 \quad \gamma_1 \quad \gamma_2 \quad \gamma_3} \qquad \underbrace{\gamma_4, \dots \gamma_{D-1}}$

$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ $\mu,\nu = 0,\dots,3$ $\qquad \gamma_a \quad a = 4,\dots,D-1$

4-D Lorentz symmetry $\otimes$ Scalar symmetry

unitary: $U(2^{(D-4)/2})$

$[\sigma_{\mu\nu}, \gamma_a] = 0$

$[\tilde{\gamma}_5, \gamma_a] = 0 \qquad \tilde{\gamma}_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$

$[H, \gamma_a] = 0 \qquad H = i\gamma_0 \bar{\nabla}\cdot\bar{\gamma}$

maximal scalar symmetry

$U_R \otimes U_L \qquad U_R = \frac{1}{2}(1+\tilde{\gamma}_5)\, U(2^{(D-4)/2})$

$U_L = \frac{1}{2}(1-\tilde{\gamma}_5)\, U(2^{(D-4)/2})$

Coleman-Mandula OK

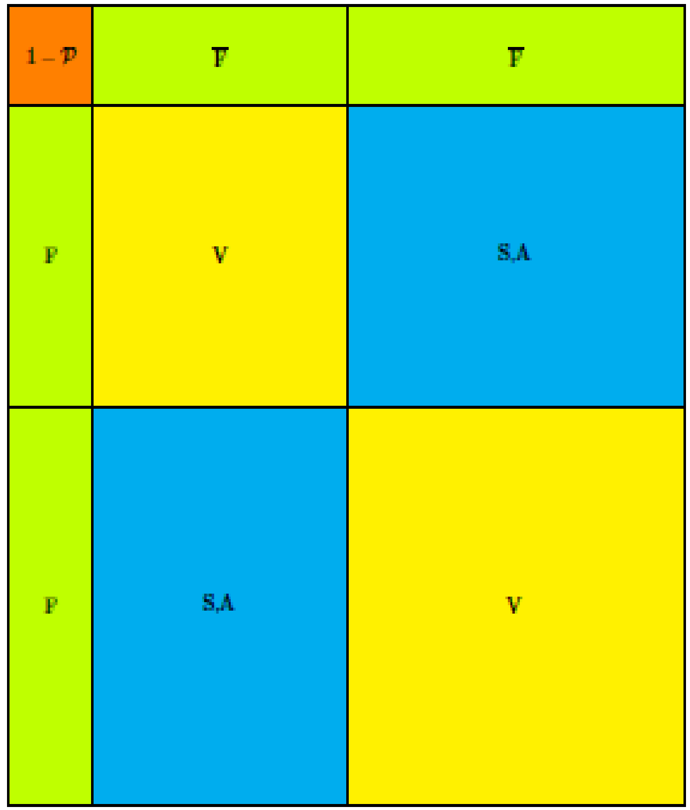


Figure 2. Representation of states in extended spin space[25], classified according to their Lorentz transformation properties: fermion (F), vector (V), scalar (S), and anti-symmetric tensor (A). Antifermions ($\bar{F}$) correspond to the Hermitian conjugate, and the blocks for V, S and A also contain antiparticle solutions.

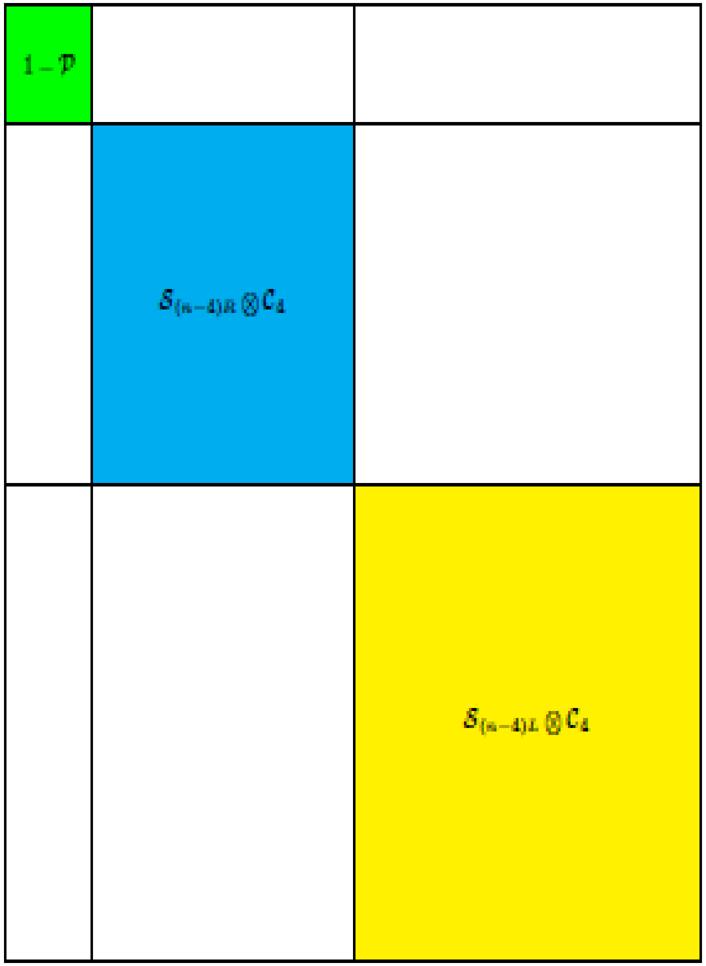


Figure 1. Schematic representation of symmetry generators in extended spin space, producing both scalar and Lorentz generators[25]

5+1-dimensions

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2\eta^{\mu\nu} \quad \mu, \nu = 0, \ldots N-1$$,

We define,

$$\tilde{\gamma}^{5} = i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3} \qquad [\tilde{\gamma}^{5}, \gamma^{\theta}] = 0$$

$$S_{N-4} = S_{(N-4)R} \times S_{(N-4)L}$$

$$S_{(N-4)R} = \tfrac{1}{2}(I + \tilde{\gamma}^{5})U(2^{(N-4)/2}) \qquad S_{(N-4)L} = \tfrac{1}{2}(I - \tilde{\gamma}^{5})U(2^{(N-4)/2}).$$

$$\mathcal{P}_S = \mathcal{P}_P = L = \frac{3}{4} - \frac{i}{4}(I + \tilde{\gamma}^5)\gamma^5\gamma^6 - \frac{1}{4}\tilde{\gamma}^5,$$

Lepton number is associated to L operator

$$SU(2)_L \times U(1)_Y$$

$$I_1 = \frac{i}{4}(1 - \tilde{\gamma}_5)\gamma^5 \,,$$

$$I_2 = -\frac{i}{4}(1 - \tilde{\gamma}_5)\gamma^6 \,,$$

$$I_3 = -\frac{i}{4}(1 - \tilde{\gamma}_5)\gamma^5\gamma^6 \,,$$

$$Y = -1 + \frac{i}{2}(1 + \tilde{\gamma}_5)\gamma^5\gamma^6 \,.$$

Electro-weak sector elements (e_R, ν_L, e_L, $W^{\pm}$, Z)

Electroweak multiplets	States Ψ	I_3	Y	Q	L	$\frac{i}{2}L\gamma^1\gamma^2$	$L\tilde{\gamma}_5$
Fermion doublet	$\nu_L^1 = \frac{1}{8}(1-\tilde{\gamma}_5)(\gamma^0+\gamma^3)(\gamma^5-i\gamma^6)$	1/2	−1	0	1	1/2	−1
	$\nu_L^2 = \frac{1}{8}(1-\tilde{\gamma}_5)(\gamma^0-\gamma^3)(\gamma^5-i\gamma^6)$	1/2	−1	0	1	−1/2	−1
	$e_L^1 = \frac{1}{8}(1-\tilde{\gamma}_5)(\gamma^0+\gamma^3)(1+i\gamma^5\gamma^6)$	−1/2	−1	−1	1	1/2	−1
	$e_L^2 = \frac{1}{8}(1-\tilde{\gamma}_5)(\gamma^0-\gamma^3)(1+i\gamma^5\gamma^6)$	−1/2	−1	−1	1	−1/2	−1
Fermion singlet	$e_R^1 = \frac{1}{8}(1+\tilde{\gamma}_5)\gamma^0(\gamma^0+\gamma^3)(\gamma^5-i\gamma^6)$	0	−2	−1	1	1/2	1
	$e_R^2 = \frac{1}{8}(1+\tilde{\gamma}_5)\gamma^0(\gamma^0-\gamma^3)(\gamma^5-i\gamma^6)$	0	−2	−1	1	−1/2	1
Scalar doublet	$\frac{1}{4\sqrt{2}}(1-\tilde{\gamma}_5)\gamma^0(1-i\gamma^5\gamma^6)$	1/2	1	1	0	0	−2
	$\frac{1}{4\sqrt{2}}(1-\tilde{\gamma}_5)\gamma^0(\gamma^5+i\gamma^6)$	−1/2	1	0	0	0	−2
Vector singlet	$\frac{1}{2\sqrt{2}}\gamma^0(\gamma^1+i\gamma^2)Y$	0	0	0	0	1	0
	$\frac{1}{2}\gamma^0\gamma^3 Y$	0	0	0	0	0	0
	$\frac{1}{2\sqrt{2}}\gamma^0(\gamma^1-i\gamma^2)Y$	0	0	0	0	−1	0
Vector triplet	$\frac{1}{8}(1-\tilde{\gamma}_5)\gamma^0(\gamma^1+i\gamma^2)(\gamma^5-i\gamma^6)$	1	0	1	0	1	0
	$\frac{1}{4\sqrt{2}}(1-\tilde{\gamma}_5)\gamma^0(\gamma^1+i\gamma^2)\gamma^5\gamma^6$	0	0	0	0	1	0
	$\frac{1}{8}(1-\tilde{\gamma}_5)\gamma^0(\gamma^1+i\gamma^2)(\gamma^5+i\gamma^6)$	−1	0	−1	0	1	0

Lagrangian constuctions

When we have interactions, free fields give more general expressions of the bosonic and fermionic fields, keeping their transformed properties

Vector field

$$A_\mu^a(x)\gamma_0\gamma^\mu I_a$$

Fermion field

$$\psi_\alpha^a(x)L^\alpha P_F M_a^F$$

Spin-extended fields can be use to build a Lagrangian formulation of the theory . For example, a interaction boson-fermion term result of vectorial term addition to the free fermionic Lagrangian.

$$\frac{1}{N_f}\mathrm{tr}\Psi^\dagger\{[i\partial_\mu I_{den} + gA_\mu^a(x)I_a]\gamma_0\gamma^\mu - M\gamma_0\}\Psi P_f$$

Supersymmetry and spin-extended model

Motivation:

SUSY and the spin-extended model have in common the general description of fermions and bosons under the same set of operators (Clifford algebras). This suggests a closer connection between the two models.

Massive particles

$\Gamma^0 := a_1 + a_1^\dagger$ $\Gamma^1 := a_2^\dagger - a_2$ $\Gamma^2 := i(a_2 + a_2^\dagger);$ $\Gamma^3 := a_1^\dagger - a_1.$

$$\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu} \qquad \Theta^{\mu\nu} := \frac{i}{4}[\Gamma^\mu, \Gamma^\nu] \qquad \widetilde{\Gamma}^5 := i\Gamma^0\Gamma^1\Gamma^2\Gamma^3$$

$$\widetilde{P}^\pm := \frac{1}{2}(Id \pm \widetilde{\Gamma^5})$$

$$\mathbf{S} = \widetilde{P}^-(\Theta^{23}, \Theta^{31}, \Theta^{12}) = \frac{i}{2}\widetilde{P}^-(\Gamma^2\Gamma^3, \Gamma^3\Gamma^1, \Gamma^1\Gamma^2),$$

5+1-dimension

$\{\Gamma^\varrho, \Gamma^\varkappa\} = 2\eta^{\varrho\varkappa}$ $\varrho, \varkappa = 0,1,2,3,5,6$

The operators Γ^5, Γ^6 y $\Gamma^5\Gamma^6$ form a SU(2) basis. We define

$$\Gamma^\pm := \frac{i}{\sqrt{2}}(\Gamma^5 \pm i\Gamma^6),$$

$$[S^3, \Gamma^\pm] = 0\,\Gamma^\pm; \quad [S^3, a_1^\dagger \Gamma^\pm] = \frac{1}{2} a_1^\dagger \Gamma^\pm;$$

$$[S^3, a_2^\dagger \Gamma^\pm] = -\frac{1}{2} a_2^\dagger \Gamma^\pm; \quad [S^3, a_2^\dagger a_1^\dagger \Gamma^\pm] = 0\, a_2^\dagger a_1^\dagger . \Gamma^\pm.$$

If we use $L = \frac{3}{4} - \frac{i}{4}(I + \widetilde{\Gamma}^5)\Gamma^5\Gamma^6 - \frac{1}{4}\widetilde{\Gamma}^5$

$$I_1 = -\frac{i}{2}L\Gamma^5$$
$$I_2 = -\frac{i}{2}L\Gamma^6$$
$$I_3 = -\frac{i}{2}L\Gamma^5\Gamma6$$
,

then we can classify:

Multilplets	States	$[S_3]$	$[I_3]$	Superpartners	$[S_3]$	$[I_3]$
Fermion doublet	$\frac{1}{4}(I - \widetilde{\Gamma}_5)a_1^\dagger(\Gamma^5 - i\Gamma^6)$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}a_2^\dagger(I - \widetilde{\Gamma}_5)a_1^\dagger(\Gamma^5 - i\Gamma^6)$	0	$\frac{1}{2}$
	$\frac{1}{4}(I - \widetilde{\Gamma}_5)a_1(\Gamma^5 - i\Gamma^6)$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}a_2(I - \widetilde{\Gamma}_5)a_1(\Gamma^5 - i\Gamma^6)$	0	$\frac{1}{2}$
Fermion singulet	$\frac{1}{4}(I + \widetilde{\Gamma}_5)a_1^\dagger(1 - i\Gamma^5\Gamma^6)$	$\frac{1}{2}$	0	$\frac{1}{4}a_1(I + \widetilde{\Gamma}_5)a_1^\dagger(1 - i\Gamma^5\Gamma^6)$	0	0
Vector triplet	$\frac{1}{2}(I - \widetilde{\Gamma}_5)\Gamma^0 a_2(\Gamma^5 - i\Gamma^6)$	1	1	$\frac{1}{2}a_1(I - \widetilde{\Gamma}_5)\Gamma^0 a_2(\Gamma^5 - i\Gamma^6)$	$\frac{1}{2}$	1
	$\frac{1}{\sqrt{2}}(I - \widetilde{\Gamma}_5)\Gamma^0 a_2(1 - i\Gamma^5\Gamma^6)$	1	0	$\frac{1}{\sqrt{2}}a_1(I - \widetilde{\Gamma}_5)\Gamma^0 a_2(1 - i\Gamma^5\Gamma^6)$	$\frac{1}{2}$	0
	$\frac{1}{2}(I - \widetilde{\Gamma}_5)\Gamma^0 a_2(\Gamma^5 + i\Gamma^6)$	1	-1	$\frac{1}{2}a_1^\dagger(I + \widetilde{\Gamma}_5)\Gamma^0 a_2(\Gamma^5 + i\Gamma^6)$	$\frac{1}{2}$	-1

CONCLUSIONS AND COMENTS

- Creation and annihilation operators algebra for N=1 is enough to reproduce the Lorentz group
- When we incorporate the ideas of the spin-extended model to the simplest non-trivial case of SUSY, we obtain the scalar group SU(2). With these, SUSY states can be classified in multiplets.
- The next step in the investigation is the consideration of interactions and a Lagrangian description model; use of the Clifford-algebra space, common to both the spin-extended model and SUSY as a guide; finally, the use of the superspace.

THANK YOU