# VSR Standard Model

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Existe la creencia de que los árboles respiran el aliento de las personas que habitan las ciudades enterradas...
Leyendas de Guatemala
Miguel Ángel Asturias,Premio Nóbel de Literatura 1967

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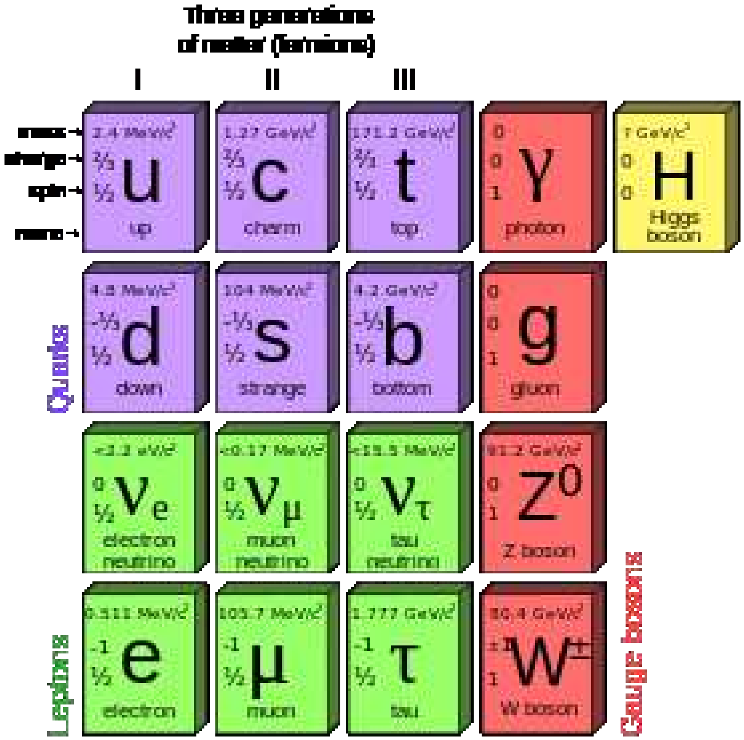



In the SM, the neutrinos are massless and have left handed quirality(L).

- Nobel Prize in Physics 2015

The proportion of each flavor in the same neutrino beam changes in time. It is proportional to the distance to the source.

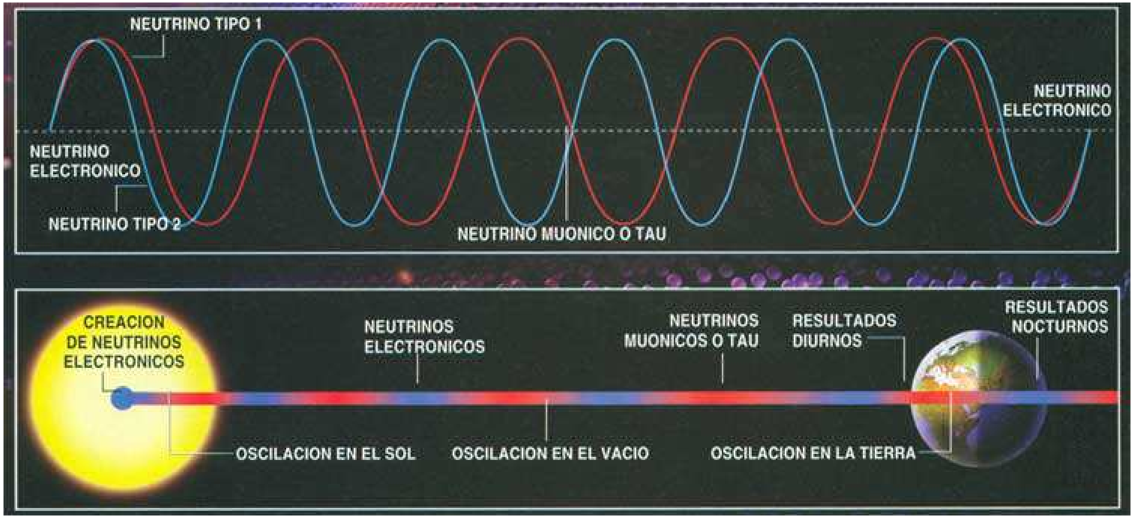



**Figure 1.**

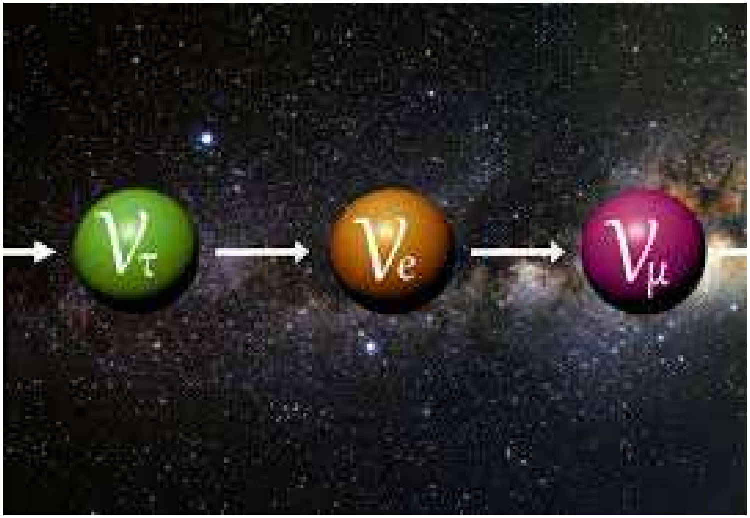



**Figure 2.**

Neutrinos can oscillate only if their mass is not zero.->Physics beyond the SM.

# Very Special Relativity

A. Cohen and S. Glashow, Phys.Rev.Lett.97:021601,2006:

All kinematical effects associated to invariance under the Lorentz group(6 parameters) can be obtained from four parameters subgroups of the Lorentz group, opening the road to new predictions which violate Lorentz symmetry, but preserve the symmetry under such subgroups.

- VSR implies special relativity (SR) in the context of local quantum field theory or of CP conservation.
- Most interesting Subgroup of the Lorentz Group:Hom(2), 3 parameters; Sim(2) , 4 parameters.

  There are no invariant tensors for these cases. So SR kinematics is preserved.

  No local Lorentz symmetry-breaking operator preserving either of these groups exists.

$$T_1 = K_x + J_y \quad T_2 = K_y - J_x$$

$$\text{Hom}(2)\text{: generators: } T_1, T_2, K_z$$

$$\text{Sim}(2)\text{: generators: } T_1, T_2, K_z, J_z$$

$$n = (1,0,0,1) \quad n.n = 0$$

$n$ is invariant under $T_1, T_2, J_z$, but under boosts in the z-direction (generated by $K_z$)

$$n \rightarrow e^{\phi} n$$

$p_1, p_2$ particle momenta.

VSR but not SR invariant: $\frac{p_1.n}{p_2.n}$

Neutrino mass in VSR:

$$\left(\not{p} - \frac{m_v^2}{2}\frac{\not{n}}{n.p}\right)\nu_L = 0, \left(\not{p} - \frac{m_v^2}{2}\frac{\not{n}}{n.p}\right)^2\nu_L =$$

$$(p^2 - m_\nu^2)\nu_L = 0$$

# Very Special Relativity Standard Model

J.A., R. Avila and P. González, Electroweak standard model with very special relativity,PHYSICAL REVIEW D 91, 105007 (2015)

- LHC do not see new particles or symmetries. It just ratifies the SM structure:particles and symmetries.
- Neutrino are massless in the SM, but in nature they are massive(neutrino oscillations).
- We want to keep the particles and symmetries of the SM, but provide masses for neutrinos
- The VSR SM is a simple theory with $SU(2)_L x U(1)_R$ symmetry, with the same number of leptons and gauge fields as in the SM.
- It is renormalizable and unitarity is preserved.
- New non-local terms that violate Lorentz invariance are able to describe in a straightforward manner the observed neutrino oscillations.
- We predict new processes such as the decay $\mu$->$e$+$\gamma$, which are forbidden in the SM.

## VSR QED

$$\mathcal{L} = \bar{\psi}\left( i\left( \not{D} + \frac{1}{2}\not{n} m^2 \left(n \cdot D\right)^{-1} \right) - M \right)\psi + \mathcal{L}_{\mathrm{ph}}$$
$$D_\mu = \partial_\mu - ieA_\mu$$

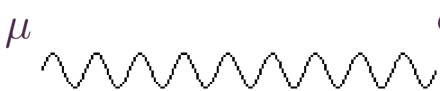
$n.n = 0$

- In the VSR SM the electron neutrino and the electron belong to a doublet under $\mathrm{SU}(2)_L$
- $m$ is the VSR mass of both electron and neutrino. After spontaneous symmetry breaking(SSB). the electron adquires an additional mass $M$. The electron mass is $M_e = \sqrt{M^2 + m^2}$.
- The neutrino mass is not affected by SSB:$M_{\nu_e} = m$.

# Feynman Rules

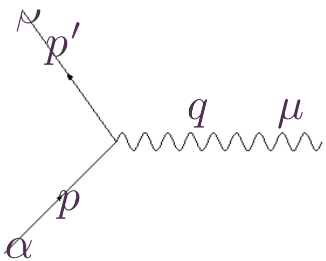

Electron propagator:

$$\frac{i\left(\not{p}+M-\frac{m^2}{2}\frac{\not{n}}{n\cdot p}\right)_{\beta\alpha}}{p^2-M^2-m^2+i\,\varepsilon}$$



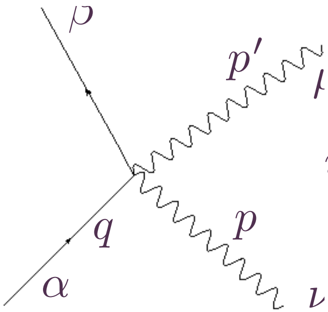

Photon propagator:

$$D_{\mu\alpha}=-\frac{n_\mu}{n.p}\frac{n_\alpha}{n.p}\frac{i\,m_G^2}{p^2-m_G^2}+\frac{n_\mu p_a+n_a p_\mu}{n.p(p^2-m_G^2)}\frac{i\,m_G^2}{p^2}\quad+$$
$$\frac{p_\mu p_\alpha}{(p^2)^2}i(1+\alpha)-\frac{i\,\eta_{\mu\alpha}}{p^2-m_G^2}$$




$$-ie(\gamma_\mu)_{\beta\alpha} + \frac{1}{2}(\not{n})_{\beta\alpha} m^2(-ei)\,(n.p)^{-1}(n_\mu)(n.p')^{-1}$$

3-vertex:




$$i(ie)^2\frac{1}{2}\not{n} m^2\, n_\mu n_\nu (n.(p'+p+q))^{-1}(n.q)^{-1}[(n.(p+q))^{-1}+(n.(p'+q))^{-1}]$$

# Mandelstam-Leibbrandt(ML)prescription

- $\int dp \frac{1}{[p^2+2p.q-m^2]^a}\frac{1}{n.p}$ has an infrared divergence when $n.p=0$
- Light-Cone gauge quantization of gauge and string theories.
- Mandelstam-Leibbrandt prescription(ML):$\frac{1}{n.p}=\lim_{\varepsilon\to 0}\frac{p.\bar{n}}{n.p p.\bar{n}+i\varepsilon}$, $\bar{n}.\bar{n}=0$, $n.\bar{n}=1$
- ML has very nice properties: The poles in the $p_0$ complex plane are situated such that the Wick's rotation from Euclidean to Minkowsky space is justified; it preserves naive power counting of loop integrals; and in gauge theories, it maintains the Ward identities of the gauge symmetry. It can be derived from Canonical Quantization.

- J.A., **Mandelstam-Leibbrandt prescription**, **Phys.Rev. D93 (2016) no.6, 065033**, Erratum: Phys.Rev. D94 (2016) no.4, 049901
- Let us compute the following simple integral:

$$A_\mu = \int dp \, \frac{f(p^2) p_\mu}{n.p}$$

where $f$ is an arbitrary function. $dp$ is the integration measure in $d$ dimensional space and $n_\mu$ is a fixed null vector($n.n = 0$). This integral is infrared divergent when $n.p = 0$.

- The ML is:

$$\frac{1}{n.p} = \lim_{\varepsilon \to 0} \frac{p.\bar{n}}{n.p p.\bar{n} + i\varepsilon} \qquad (1)$$

where $\bar{n}_\mu$ is a new null vector with the property $n.\bar{n} = 1$.

- To compute $A_\mu$ we have to know the specific form of $f$, provide an specific form of $n_\mu$ and $\bar{n}_\mu$, and evaluate the residues of all poles of $\frac{f(p^2)}{n.p}$ in the $p_0$ complex plane, a rather formidable task for an arbitrary $f$.

- Instead we want to point out the following symmetry:

$$n_\mu \to \lambda n_\mu, \bar{n}_\mu \to \lambda^{-1}\bar{n}_\mu, \lambda \neq 0, \lambda \varepsilon R \tag{2}$$

- It preserves the definitions of $n_\mu$ and $\bar{n}_\mu$:

$$\begin{aligned} 0 = n.n &\to \lambda^2 n.n = 0 \\ 0 = \bar{n}.\bar{n} &\to \lambda^{-2}\bar{n}.\bar{n} = 0 \\ 1 = n.\bar{n} &\to n.\bar{n} = 1 \end{aligned}$$

- We see from (1) that:

$$\frac{1}{n.p} \to \frac{1}{n.p}\lambda^{-1}$$

- Now we compute $A_\mu$, based on its symmetries. It is a Lorentz vector which scales under (2) as $\lambda^{-1}$. The only Lorentz vectors we have available in this case are $n_\mu$ and $\bar{n}_\mu$. But (2) forbids $n_\mu$. That is:

$$A_\mu = a\,\bar{n}_\mu$$

- Multiply by $n_\mu$ to find $A.n = a$. Thus $a = \int dp\, f(p^2)$. Finally:

$$\int dp\,\frac{f(p^2)p_\mu}{n.p} = \bar{n}_\mu \int dp\, f(p^2)$$

- By the same token we find

$$A_{\mu\nu\lambda} = \int dp\,\frac{f(p^2)p_\mu p_\nu p_\lambda}{n.p} = a(\bar{n}_\mu g_{\nu\lambda})_S + b(\bar{n}_\mu \bar{n}_\nu n_\lambda)_S$$

  where $()_S$ means symmetric in all Lorentz indices.

- We get:

$$A_{\mu\nu\lambda}n^\lambda = \frac{1}{d} g_{\mu\nu}\int dp\, f(p^2)p^2 = a(\bar{n}_\mu n_\nu + \bar{n}_\nu n_\mu + g_{\mu\nu} n.\bar{n}) + b(n.\bar{n}\bar{n}_\nu n_\mu + n.\bar{n}\bar{n}_\mu n_\nu)$$

$$a + b n.\bar{n} = 0, a = \frac{1}{d}\int dp\, f(p^2)p^2$$

The integrals on $p_\mu$ are dimensionally regularized.

- Therefore:

$$\int dp\,\frac{f(p^2)p_\mu p_\nu p_\lambda}{n.p} = \frac{1}{d}\int dp\, f(p^2)p^2\{(\bar{n}_\mu g_{\nu\lambda})_S - (\bar{n}_\mu \bar{n}_\nu n_\lambda)_S\}$$

- We consider now a more general integral. We will see here that regularity of the answer will determine it uniquely.

  Consider:

$$A = \int dp \, \frac{F(p^2, p.q)}{n.p} = \bar{n}.q \, f(q^2, n.q \, \bar{n}.q) \tag{3}$$

  $q_\mu$ is an external momentum, a Lorentz vector. $F$ is an arbitrary function. The last relation follows from (2), for a certain $f$ we will find in the following.

- We get

$$\frac{\partial A}{\partial q^\mu} = \int dp \, \frac{F_{,u} p_\mu}{n.p} =$$
$$\bar{n}_\mu f(x,y) + 2\bar{n}.q q_\mu \frac{\partial}{\partial x} f(x,y) + [(\bar{n}.q\,)^2 n_\mu + n.q \, \bar{n}.q \, \bar{n}_\mu] \frac{\partial}{\partial y} f(x,y)$$

  We defined $u = p.q$,$x = q^2$,$y = n.q \, \bar{n}.q$ . $()_{,u}$ means derivative respects to $u$.

$$\frac{\partial A}{\partial q^\mu} n_\mu = \int dp \, F_{,u} = \quad g(x) =$$
$$f(x,y) + 2y \frac{\partial}{\partial x} f(x,y) + y \frac{\partial}{\partial y} f(x,y) \tag{4}$$

- Assuming that the solution and its partial derivatives are finite in the neighborhood of $y = 0$, it follows from the equation that $f(x, 0) = g(x)$. That is the partial differential equation has a unique regular solution.
- We can find the solution of (4) using the method of characteristics.

1. Using dimensional regularization, we obtain:

$$\int dp \frac{1}{[p^2+2p.q-m^2]^a}\frac{1}{(n.p)^b} = (-1)^{a+b} i(\pi)^\omega (-2)^b \frac{\Gamma(a+b-\omega)}{\Gamma(a)\Gamma(b)} (\bar{n}.q)^b \int_0^1 dt\, t^{b-1} \frac{1}{(m^2+q^2-2n.q\,\bar{n}.qt)^{a+b-\omega}}, \quad \omega = d/2 \qquad (5)$$

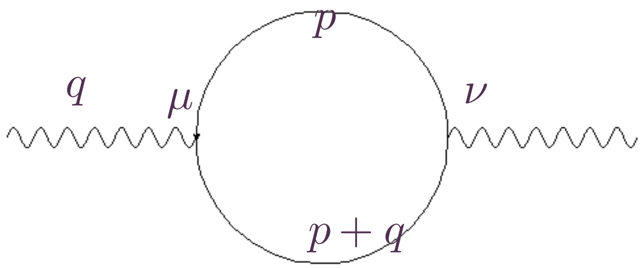
2. Other integrals can be obtained deriving respects to $q_\mu$:

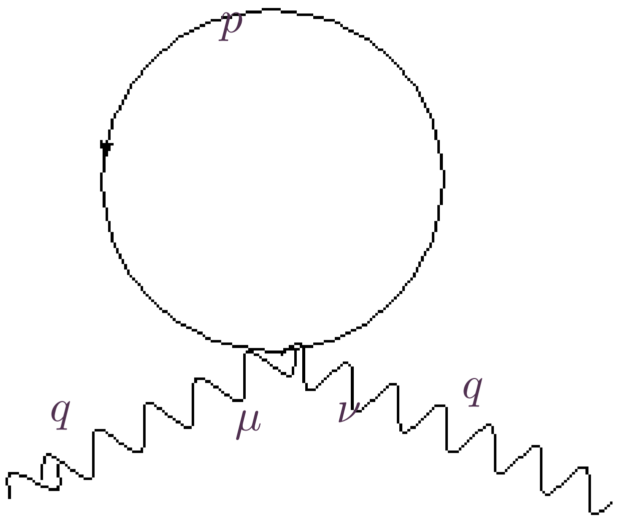
$$\int dp \frac{p_\mu}{[p^2+2p.q-m^2]^{a+1}}\frac{1}{(n.p)^b} = (-1)^{a+b} i(\pi)^\omega (-2)^{b-1} \frac{\Gamma(a+b-\omega)}{\Gamma(a+1)\Gamma(b)} (\bar{n}.q)^{b-1} b\, \bar{n}_\mu \int_0^1 dt\, t^{b-1} \frac{1}{(m^2+x-2yt)^{a+b-\omega}} + (-1)^{a+b} i(\pi)^\omega (-2)^b \frac{\Gamma(a+b+1-\omega)}{\Gamma(a+1)\Gamma(b)} (\bar{n}.q)^b \int_0^1 dt\, t^{b-1} \frac{q_\mu - t(n.q\bar{n}_\mu + \bar{n}.q n_\mu)}{(m^2+x-2yt)^{a+b+1-\omega}} \qquad (6)$$

- ML does not preserve $\mathrm{Sim}(2)$ symmetry. It requieres a second fixes null vector $\bar{n}$.
- It is possible to modify ML such that it is compatible with $\mathrm{Sim}(2)$ symmetry.(J.A. to be published)
- $$\int dp \frac{1}{[p^2+2p.q-m^2]^a}\frac{1}{(n.p)^b} =$$
$$(-1)^{a+b} i(\pi)^{\omega}(-2)^b \frac{\Gamma(a+b-\omega)}{\Gamma(a)\Gamma(b)}(\bar{n}.q)^b \int_0^1 dt\, t^{b-1}\frac{1}{(m^2+x-2yt)^{a+b-\omega}}, \quad \omega = d/2$$
- We trade $\bar{n}_\mu$ by $q_\mu$. i.e. $\bar{n}_\mu = a\, n_\mu + b\, q_\mu$. From the conditions: $\bar{n}.\bar{n}=0$,$\bar{n}.n=1$ we get $\bar{n}_\mu = -\frac{q^2}{2\,(n.\,q)^2}\, n_\mu + \frac{q_\mu}{n.\,q}$.
- Therefore, for instance,
$$\int dp \frac{1}{[p^2+2p.q-m^2]^a}\frac{1}{(n.p)^b} =$$
$$(-1)^{a+b} i(\pi)^{\omega}(-2)^b \frac{\Gamma(a+b-\omega)}{\Gamma(a)\Gamma(b)}\left(\frac{q^2}{2n.q}\right)^b \int_0^1 dt\, t^{b-1}\frac{1}{(m^2+q^2(1-t))^{a+b-\omega}}, \quad \omega = d/2$$

  Notice that now we respect the $Sim(2)$ invariance of the original integral.
- In the following we use the modified ML prescription.

# Vacuum Polarization



$$i\,\Pi_{\mu\nu} = -\left(-q^2\frac{n_\mu n_\nu}{(n.q)^2} + \frac{n_\mu q_\nu + n_\nu q_\mu}{n.q} - \eta_{\mu\nu}\right)4m^2e^2 i(4\pi)^{-2}\int_0^1 \frac{dx}{(1-x)} \log\left[1 - \frac{q^2(1-x)^2}{m^2+M^2-q^2(1-x)x}\right] +$$

$$(-ie)^2(\eta_{\mu\nu}q^2 - q_\mu q_\nu)\frac{i}{(4\pi)^\omega}\int_0^1 dx\Gamma(2-\omega)\frac{8x(1-x)}{\Delta^{2-\omega}}$$

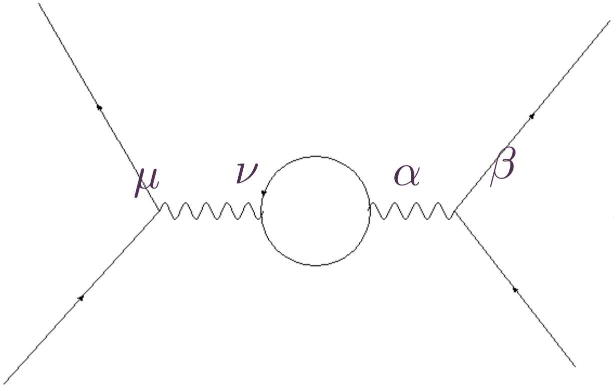
$$\Delta = M^2 + m^2 - (1-x)x\,q^2$$

Computing the finite part and normalizing at $q^2 = 0$, we get:

$$i\,\Pi_{\mu\nu} = -\left(-q^2\frac{n_\mu n_\nu}{(n.q)^2} + \frac{n_\mu q_\nu + n_\nu q_\mu}{n.q} - \eta_{\mu\nu}\right)m^2 i\frac{e^2}{4\pi^2}\int_0^1 \frac{dx}{(1-x)} \log\left[1 - \frac{q^2(1-x)^2}{m^2+M^2-q^2(1-x)x}\right] +$$

$$i(\eta_{\mu\nu}q^2 - q_\mu q_\nu)\frac{e^2}{2\pi^2}\int_0^1 dx\,x(1-x)\log\left(\frac{M^2+m^2-(1-x)x\,q^2}{M^2+m^2}\right)$$

Ward Identity: $q_\mu \Pi_{\mu\nu} = 0$



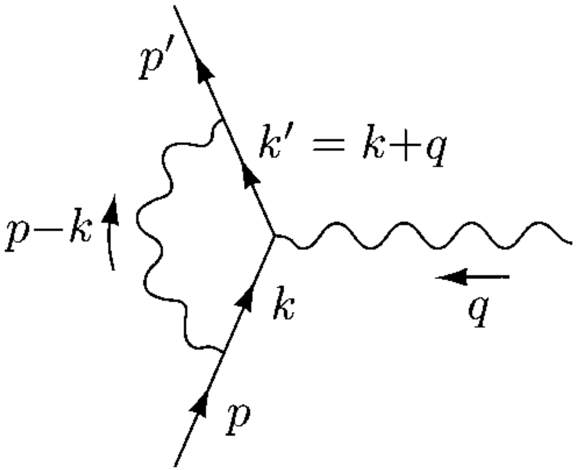

$$j^\mu(q)D_{\mu\nu}\Pi_{\nu\alpha}D_{\alpha\beta}j^\beta(-q)$$

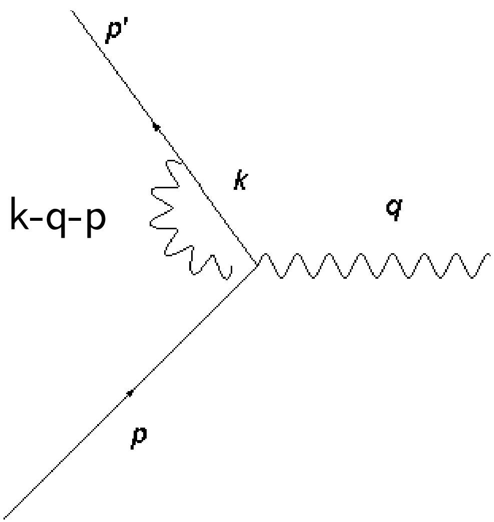
- Effective Potential $V(r)=\frac{\bar{e}_1\bar{e}_2}{(2\pi)^3}\int d^3\mathrm{q}\mathrm{e}^{i\,\vec{q}\cdot\vec{r}}\bar{V}(\vec{q})\left(1+\frac{1}{2}m^2\frac{e^2}{4\pi^2}\frac{1}{M_e^2}\right)$
- Additional contribution to the delta potential: $-\frac{4}{15}\frac{\alpha^2}{M_e^2}\left(1+\frac{m^2}{M_e^2}\frac{75}{24}\right)$
- From the bound on the fine structure

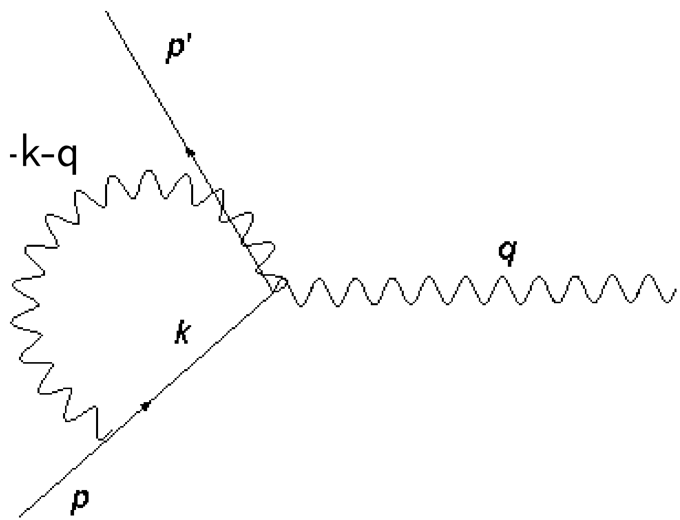
$$\frac{\Delta\alpha}{\alpha}\simeq \frac{m^2}{M_e^2}\frac{75}{48}\leqslant 10^{-8}$$

$$\frac{m}{M}\lesssim 10^{-4}$$

# Vertex Correction



# Form Factors in the NR limit(Work in progress)

| | | |
|---|---|---|
| $2M_e\varphi_s^{\dagger}\varphi_s A_0$ | $F_1$ | $F_1$ order 0 and 1 |
| $\frac{3m^2}{4M^2n_0} i\varepsilon_{ijk}\varphi_s^{\dagger}\sigma^i\varphi_{s'}n_j q_k A_0$ | $F_1(0)$ | |
| $i\varepsilon_{ijk}q_j\varphi_s^{\dagger}\sigma^k\varphi_{s'}A_i$ | $F_1(0)$ | |
| $\frac{m^2}{2n_0^2M^2}\left(-in_i\varphi_s^{\dagger}\sigma^a\varphi_{s'}\varepsilon_{abc}n_b q_c + i\varepsilon_{ijk}\varphi_s^{\dagger}\sigma^j\varphi_{s'}n_k\vec{n}.\vec{q}\,\right)A_i$ | $F_1(0)$ | |
| $-2in_0\,M\varepsilon_{ijk}\varphi_s^{\dagger}\sigma^k\varphi_{s'}q_j A_i$ | $G_2(0)$ | |
| $-i\varepsilon_{ijk}n_k\frac{m^2}{M}\varphi_s^{\dagger}\hat{n}.\vec{\sigma}\varphi_{s'}q_j A_i$ | $G_2(0)$ | |
| $-i\left(-2M\varepsilon_{ijk}\,n_k\varphi_s^{\dagger}\sigma^j\varphi_{s'} - 2M_e\,in_i\varphi_s^{\dagger}\varphi_{s'}\right)A_0\,q_i$ | $G_2(0)$ | |
| $2M_e\,n_0\varphi_s^{\dagger}\varphi_{s'}Q_\mu A^\mu$ | $G_3(0)$ | |
| $\left(-4M_e\varepsilon_{ijk}n_k\varphi_s^{\dagger}\vec{n}.\vec{\sigma}\varphi_{s'} + 4M_e\vec{n}.\vec{n}\varepsilon_{ijk}\varphi_s^{\dagger}\sigma^k\varphi_{s'}\right)q_j A_i$ | $F_3(0)$ | |
| $4M_e\,n_0\varepsilon_{ijk}n_j\varphi_s^{\dagger}\sigma^k\varphi_{s'}A_0 q_i$ | $F_3(0)$ | |
| $i\varepsilon_{ijk}\varphi_s^{\dagger}\sigma^k\varphi_{s'}A_i q_j$ | $F_2(0)$ | |
| $-i\frac{m^2}{2M^2\,n_0}\varepsilon_{ijk}n_j\varphi_s^{\dagger}\sigma^k\varphi_s A_0 q_j$ | $F_2(0)$ | |

# Conclusions and open problems

- We applied the VSR formalism to the Standard Model. This modification admits the generation of a neutrino mass without lepton number violation and without sterile neutrinos or another types of additional particles.
- Now we have non local mass terms that violate Lorentz invariance.
- The model is renormalizable and unitarity of the $S$ matrix is preserved.
- We study the QED part of the VSR SM.Feynman rules are obtained
- We invented a $\mathrm{Sim}(2)$ invariant dimensional regularization
- We computed the vacuum polarization graphs. They satisfy the Ward identity
- Bounds on the mass on the neutrino are obtained.$\frac{m}{M_e} \lesssim 10^{-4}$
- More bounds on the mass of the neutrino from new form factors and the anomalous magnetic moment of the electron. Work in progress.
- Euler-Heisenberg Lagrangian?. The null vector $n_\mu$ should appear explicitly->anisotropy
- Anisotropic propagation of photons in the Universe?

THANK YOU!