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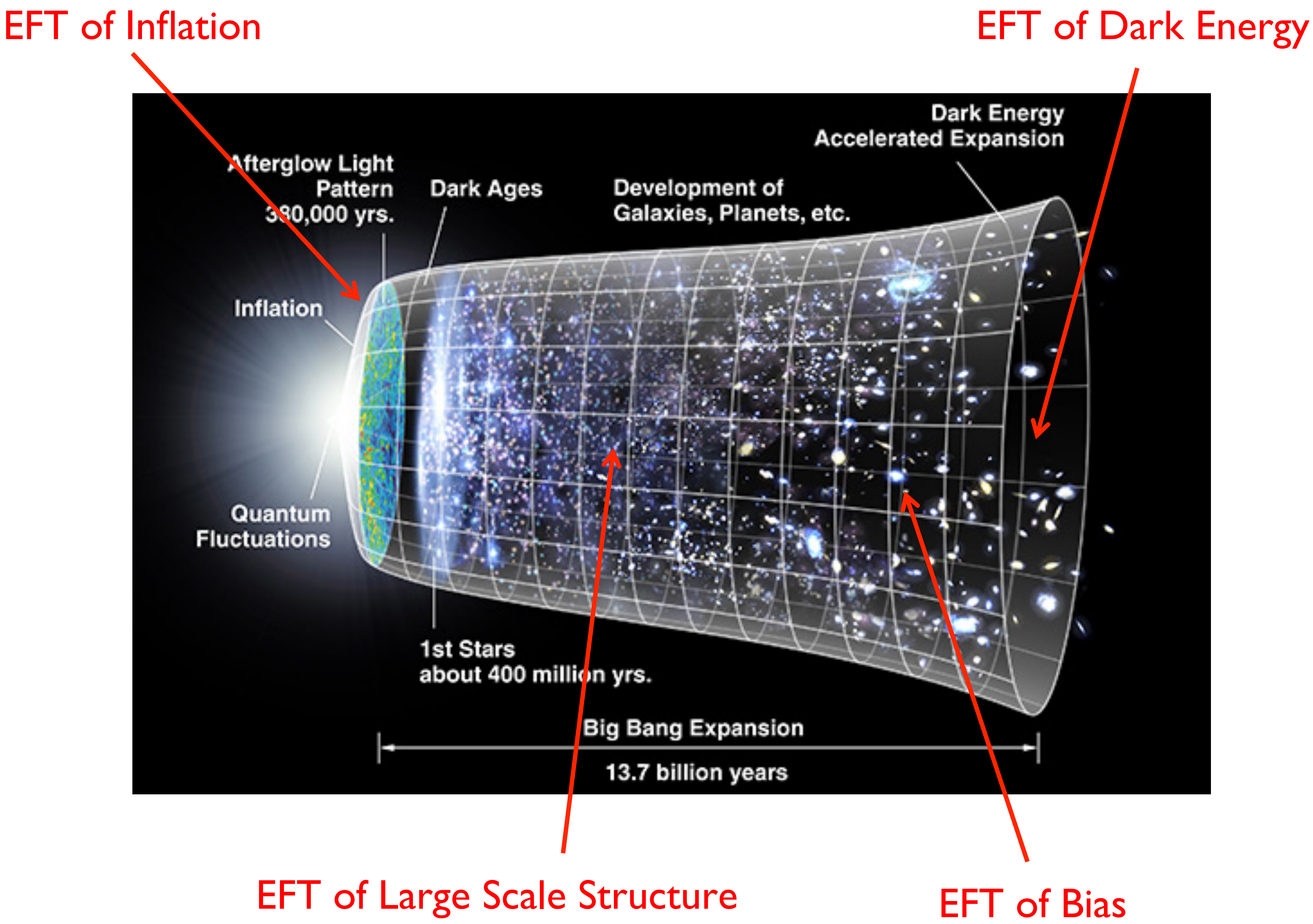
EFT in Cosmology

SILAFAE 2016

EFT in cosmology



EFT in cosmology

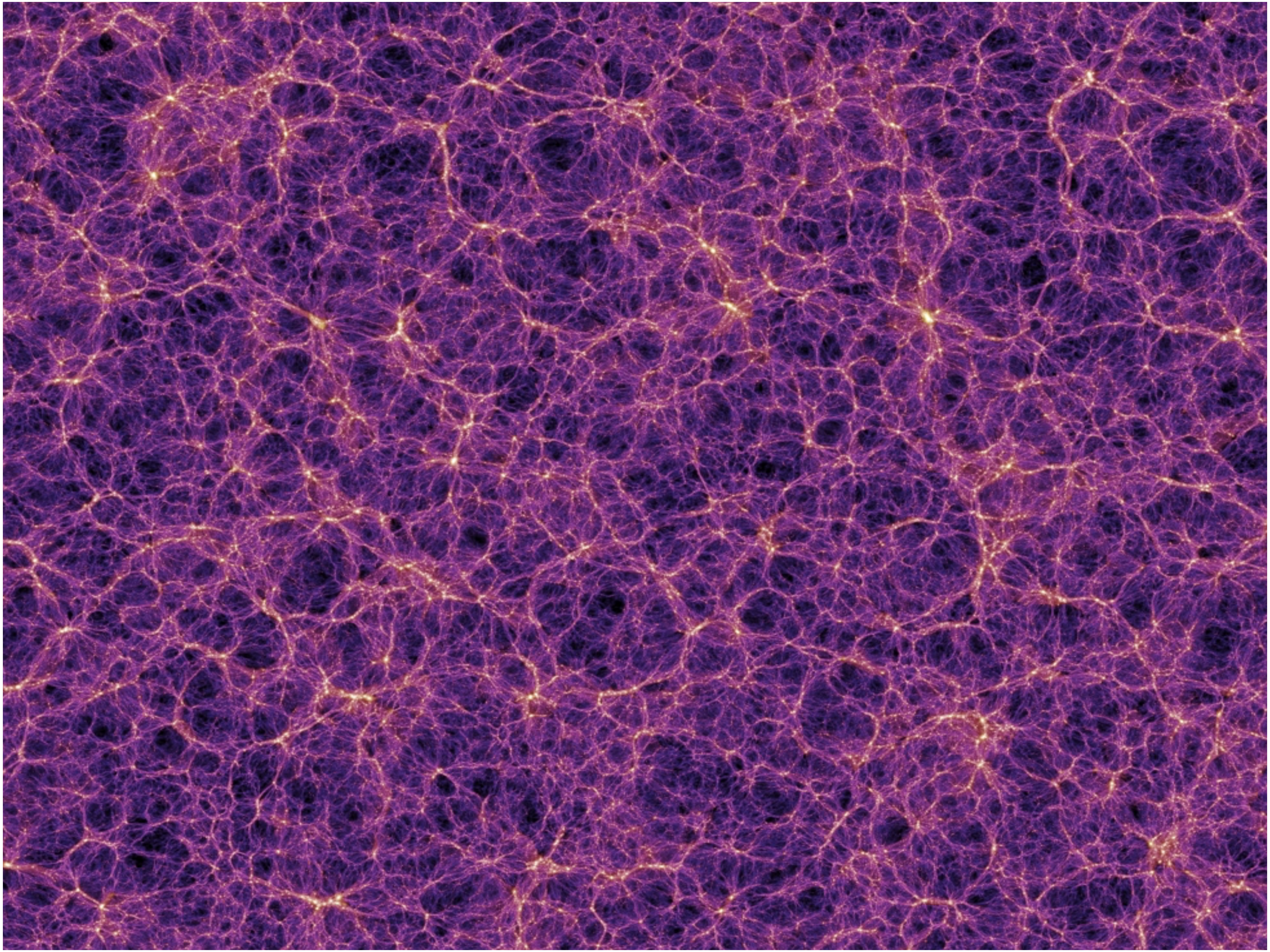
Prolification of EFT tecniques in cosmology. Why?

Better and better data!

E.g. Before year 2000 $\Omega_K < 1$, now $\Omega_K < 0.005$

1. Explore all possible models
2. Parametrize needed precision
3. Parametrize uncalculable effects

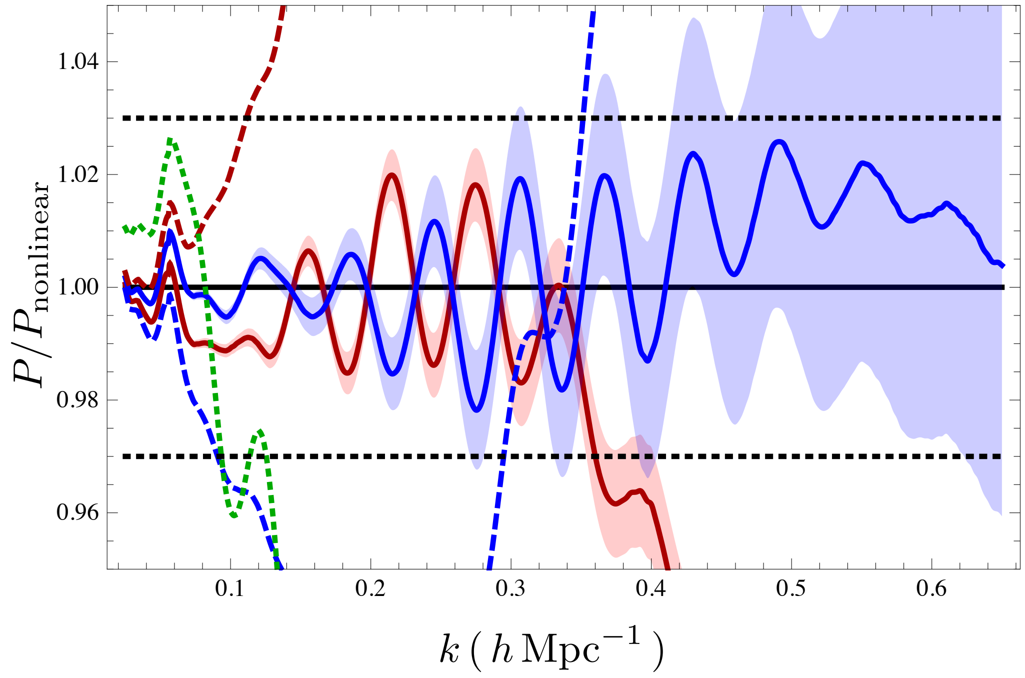
Separation of scales



EFT of Large Scale Structure

Baumann, Nicolis, Senatore, Zaldarriaga 10

Describe perturbatively clustering of DM



Continuity + Euler equations

$$\partial_\tau \delta_l + \partial_i \left[(1+\delta_l)\, v_l^i\right] = 0$$

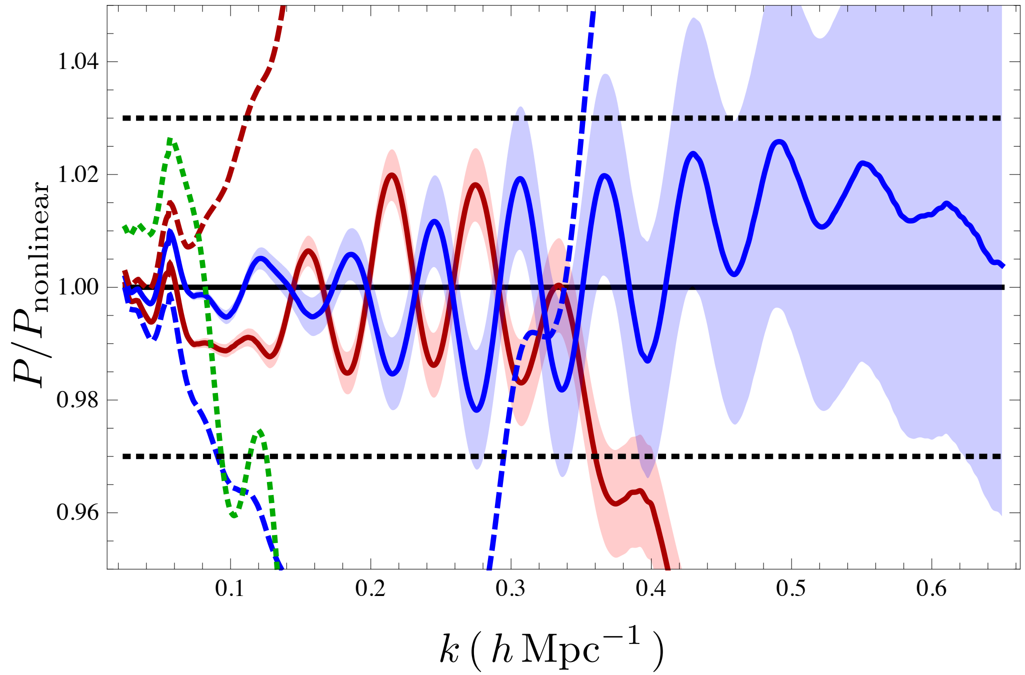
$$\partial_\tau v_l^i + \mathcal{H} v_l^i + \partial_i \phi + v_l^k \partial_k v_l^i = 0$$

Non-linearities: SPT

EFT of Large Scale Structure

Baumann, Nicolis, Senatore, Zaldarriaga 10

Describe perturbatively clustering of DM



Continuity + Euler equations

New terms integrating out short modes

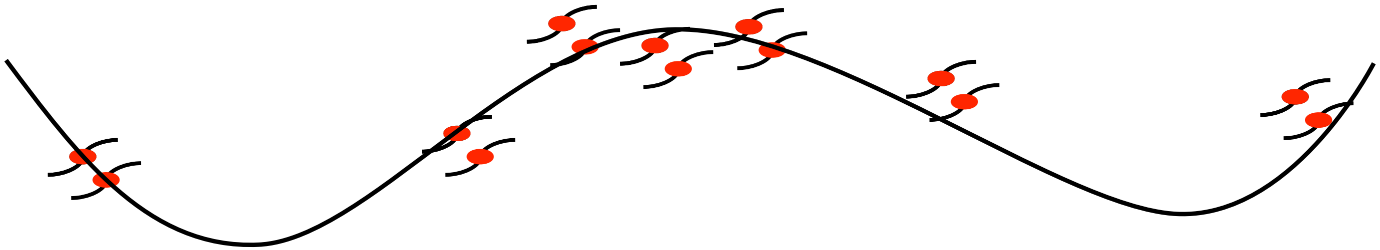
$$\partial_\tau \delta_l + \partial_i \left[(1+\delta_l)\, v_l^i \right] = 0\,,$$

$$\partial_\tau v_l^i + \mathcal{H} v_l^i + \partial_i \phi + v_l^k \partial_k v_l^i = -c_s^2 \partial^i \delta_l + \frac{3}{4}\frac{c_{sv}^2}{\mathcal{H}} \partial^2 v_l^i + \frac{4c_{bv}^2 + c_{sv}^2}{4\mathcal{H}} \partial^i \partial_j v_l^j - \Delta J^i \ldots$$

EFT of Bias

Mc Donald 06

How to relate distributions of tracers (clusters, Galaxies) to the underlying mass distribution?



$$\delta_h(\vec{x},t) = b_1\delta(\vec{x},t) + b_2\delta^2(\vec{x},t) + \ldots + b_{\nabla^2}\nabla^2\delta(\vec{x},t) + \ldots + (\partial_i\partial_j\Phi)^2 + \ldots$$

- Separation of scales object/perturbation
- All operators compatible with symmetries (e.g. equivalence principle)
- Composite operators, renormalization....

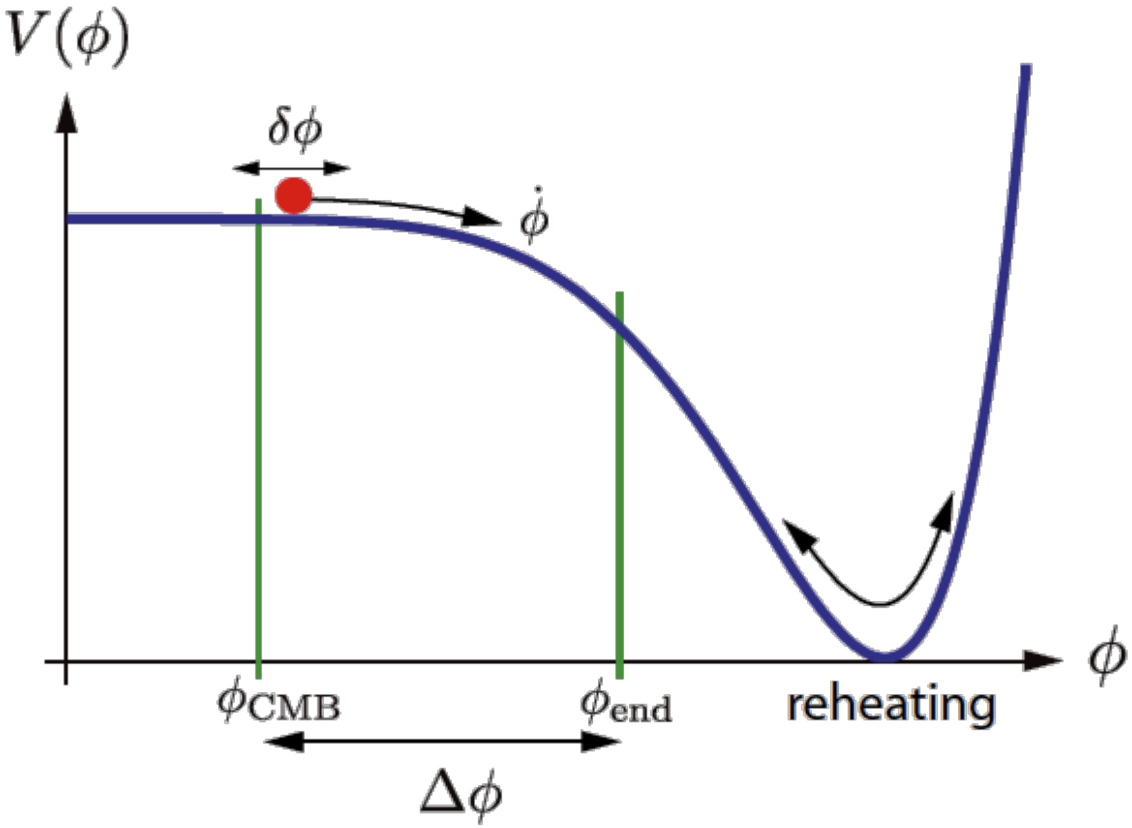
Slow-roll inflation

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

Curvature redshifts away during inflation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$



$$\ddot{\phi} + \underline{3H\dot{\phi}} + V'(\phi) = 0$$

friction is dominant

$$\epsilon = \frac{1}{2}M_P^2\left(\frac{V'}{V}\right)^2$$

$$\eta = M_P^2\frac{V''}{V}$$

$$\epsilon, \eta, \ldots \ll 1$$

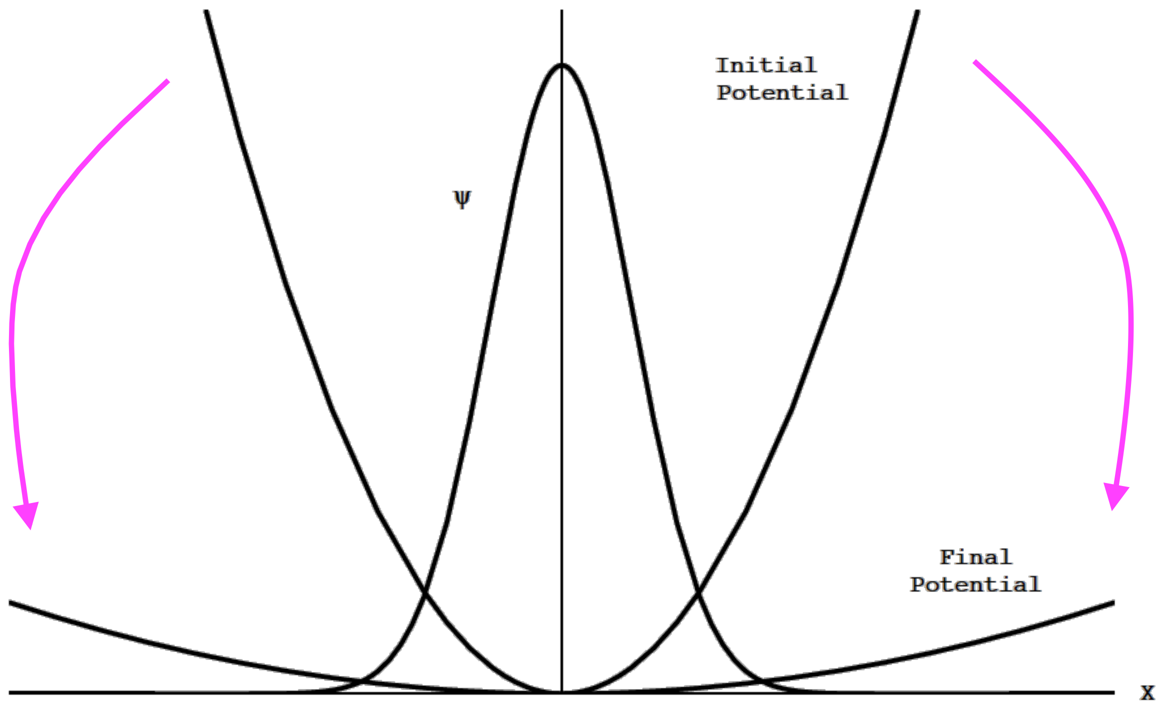
$\hbar \neq 0$

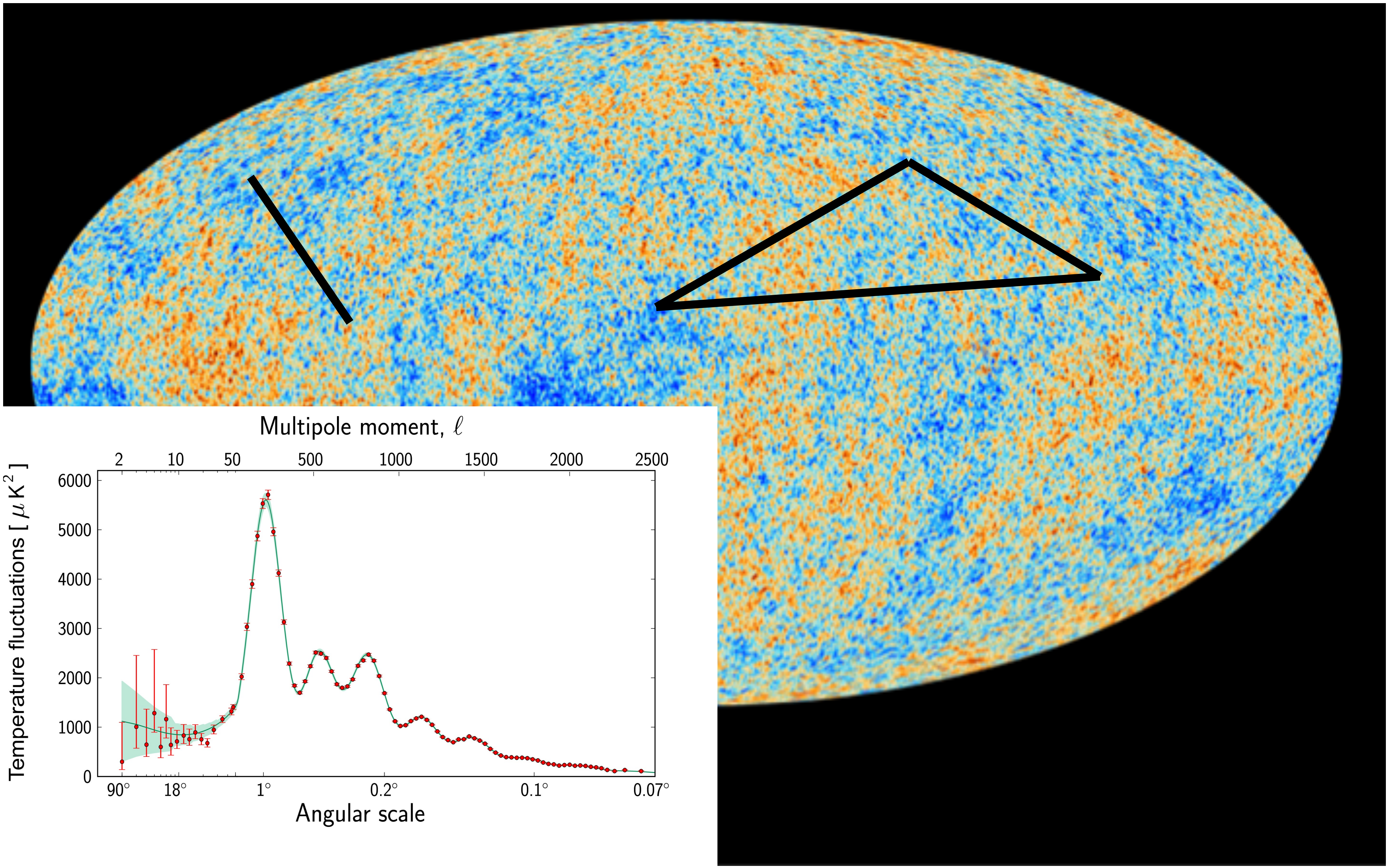
Each inflaton Fourier mode behaves as a harmonic oscillator with time dependent parameters

$$S = \frac{1}{2}\int dt d^3x \frac{\dot{\phi}^2}{H^2}\left[a^3\dot{\zeta}^2 - a(\partial\zeta)^2\right]$$

$$\delta\phi = 0$$

$$h_{ij} = a^2(t)\left[e^{2\zeta}\delta_{ij} + \gamma_{ij}\right]$$



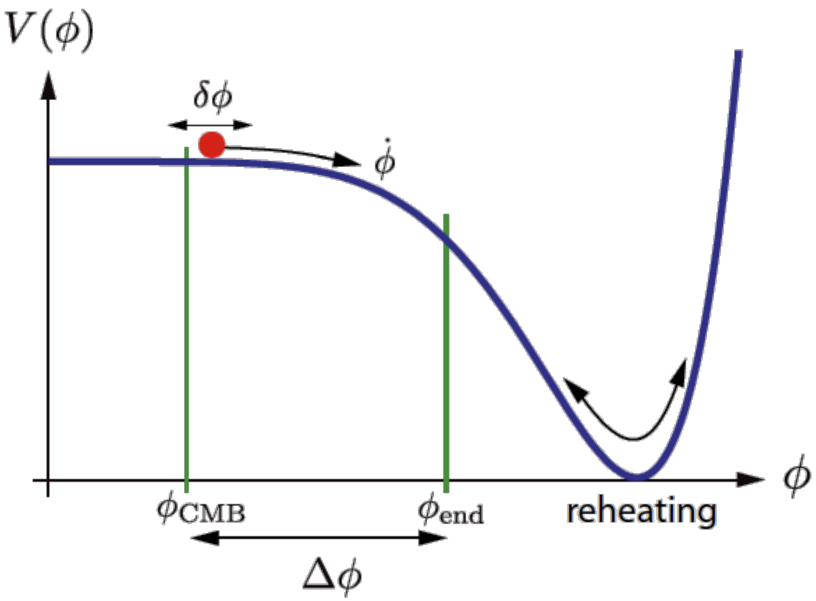


$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle \qquad \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle \qquad \langle \gamma_{\vec{k}_1} \gamma_{\vec{k}_2} \rangle \qquad \langle \gamma_{\vec{k}_1} \gamma_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle \qquad \dots$$

We were born Gaussian

$$NG \sim f_{\rm NL} \cdot P_\zeta^{1/2} \lesssim 3 \cdot 10^{-4}$$

Slow-roll = weak coupling = Gaussianity



$$\epsilon = \frac{1}{2} M_P^2 \left(\frac{V'}{V}\right)^2$$

$$\eta = M_P^2 \frac{V''}{V}$$

$$\epsilon, \eta, \ldots \ll 1$$

$$\lambda \equiv V^{(4)} \lesssim \mathcal{O}(\epsilon^3, \eta^3)(10^{-5})^2$$

Compare with Higgs: $\lambda \sim 0.12$

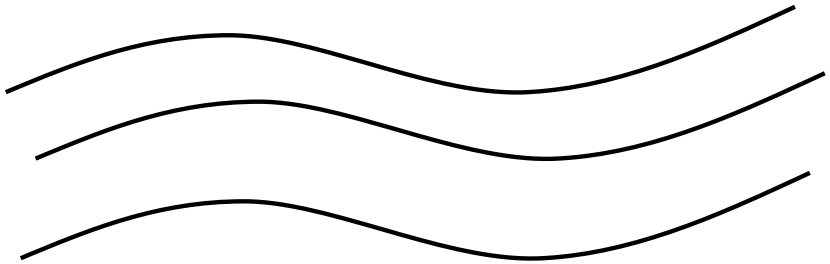
Effective Field Theory of Inflation

PC, Luty, Nicolis, Senatore 06
Cheung, PC, Fitzpatrick, Kaplan, Senatore 07

EFT:

- Identify relevant degrees of freedom: focus on single-field inflation
- Identify symmetries at play

Privileged slicing



t = const

$$\cancel{t \to t + \xi^0(t, \vec{x})} \qquad\qquad x^i \to x^i + \xi^i(t, \vec{x})$$

- Write lowest dim. operators compatible with symmetries

Effective Field Theory of Inflation

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_P^2 R \right.$$

Fully diff. invariant

Effective Field Theory of Inflation

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_P^2 R - c(t) g^{00} - \Lambda(t) \right.$$

g^{00} is invariant under spatial diffs.

Effective Field Theory of Inflation

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_P^2 R + M_P^2 \dot{H} g^{00} - M_P^2 (3H^2 + \dot{H}) \right.$$

coefficients are fixed by background

Effective Field Theory of Inflation

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_P^2 R + M_P^2 \dot{H} g^{00} - M_P^2(3H^2 + \dot{H}) + \frac{M_2(t)^4}{2!}(g^{00}+1)^2 + \frac{M_3(t)^4}{3!}(g^{00}+1)^3 + \ldots\right]$$

Quadratic, cubic etc operators only affect perturbations at given order

Effective Field Theory of Inflation

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}M_P^2R+M_P^2\dot{H}g^{00}-M_P^2(3H^2+\dot{H})+\frac{M_2(t)^4}{2!}(g^{00}+1)^2\right.$$
$$\left.+\frac{M_3(t)^4}{3!}(g^{00}+1)^3+\ldots+\hat{M}^3(t)(g^{00}+1)\delta K_\mu^{\ \mu}-\frac{\bar{M}_2(t)^2}{2}\delta K_\mu^{\ \mu\,2}+\ldots\right]$$

Higher derivative corrections suppressed by $\dfrac{\partial}{\Lambda}$ (unless extra symmetry)

All the models

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_P^2 R + \underline{M_P^2 \dot{H} g^{00} - M_P^2(3H^2+\dot{H})} + \underline{\frac{M_2(t)^4}{2!}(g^{00}+1)^2}\right.$$
$$\left.+\underline{\frac{M_3(t)^4}{3!}(g^{00}+1)^3} + \ldots + \underline{\hat{M}^3(t)(g^{00}+1)\delta K^\mu_\mu - \frac{\bar{M}_2(t)^2}{2}\delta K^{\mu\;2}_\mu} + \ldots\right]$$

- Standard slow-roll: $-\frac{1}{2}(\partial\phi)^2 - V(\phi) \quad \longrightarrow \quad -\frac{\dot{\phi}_0(t)^2}{2}g^{00} - V(\phi_0(t))$

- K-inflation: $P(X,\phi), \quad X \equiv (\partial\phi)^2$

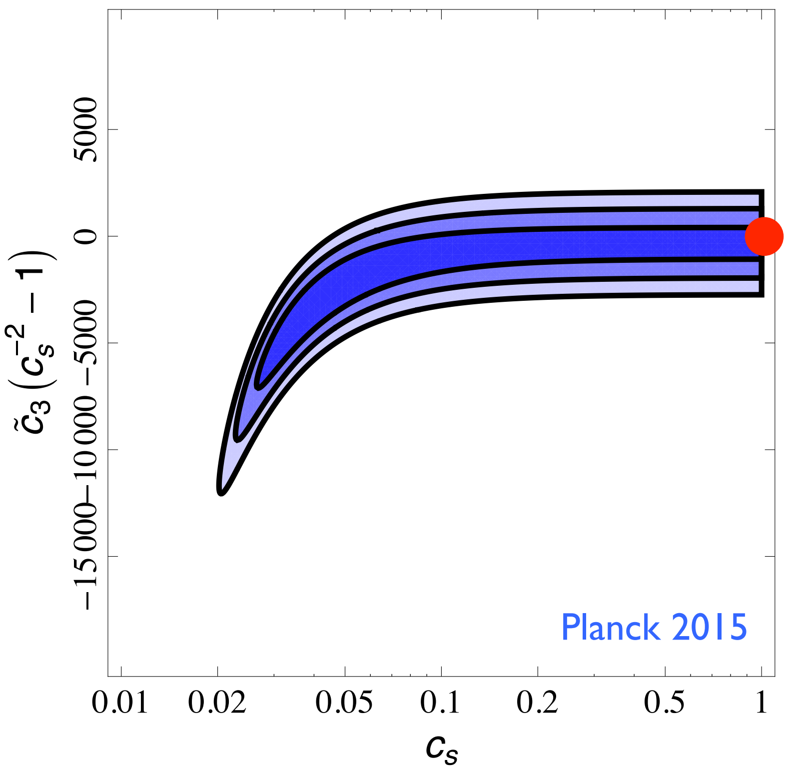
- Galileon inflation, Ghost inflation

Decoupling limit

At high energy the Goldstone decouples (like longitudinal Ws): usually good for inflation

$$\left[-M_P^2\dot H\left(\dot\pi^2-\frac{(\partial_i\pi)^2}{a^2}\right)+2M_2^4\left(\dot\pi^2+\dot\pi^3-\dot\pi\frac{(\partial_i\pi)^2}{a^2}\right)-\frac{4}{3}M_3^4\dot\pi^3\right]$$

$$=\left[-M_P^2\dot H\left(\dot\pi^2-\frac{(\partial_i\pi)^2}{a^2}\right)-M_P^2\dot H(1-c_s^{-2})\dot\pi\frac{(\partial_i\pi)^2}{a^2}+\left(M_P^2\dot H(1-c_s^{-2})-\frac{4}{3}M_3^4\right)\dot\pi^3\right]$$



Analogue of precision EW observables

Relevant target: $f_{NL}^{EQ}\sim 1$ $c_s\sim 1$

Present and future of NGs

- $f_{NL}^{eq} < 43$ Th. target: $f_{NL}^{eq} \sim 1$

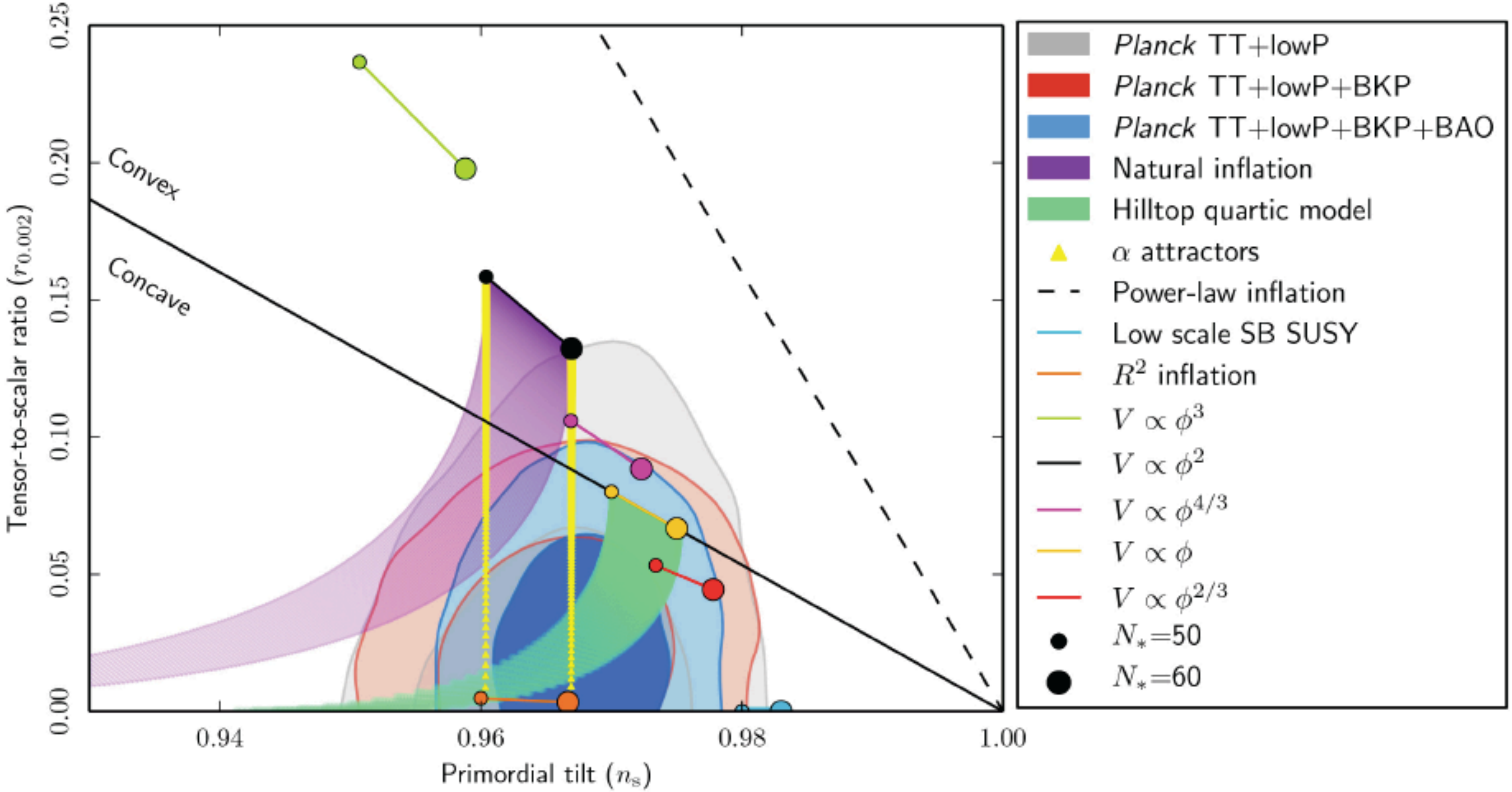
Very difficult to achieve on a 10y timescale

- $f_{NL}^{loc} < 5$ Th. target: $f_{NL}^{loc} \sim 1$
  (second-field mechanism usually gives $f_{NL}^{loc} > 1$)

Possible (?) with next generation LSS probes

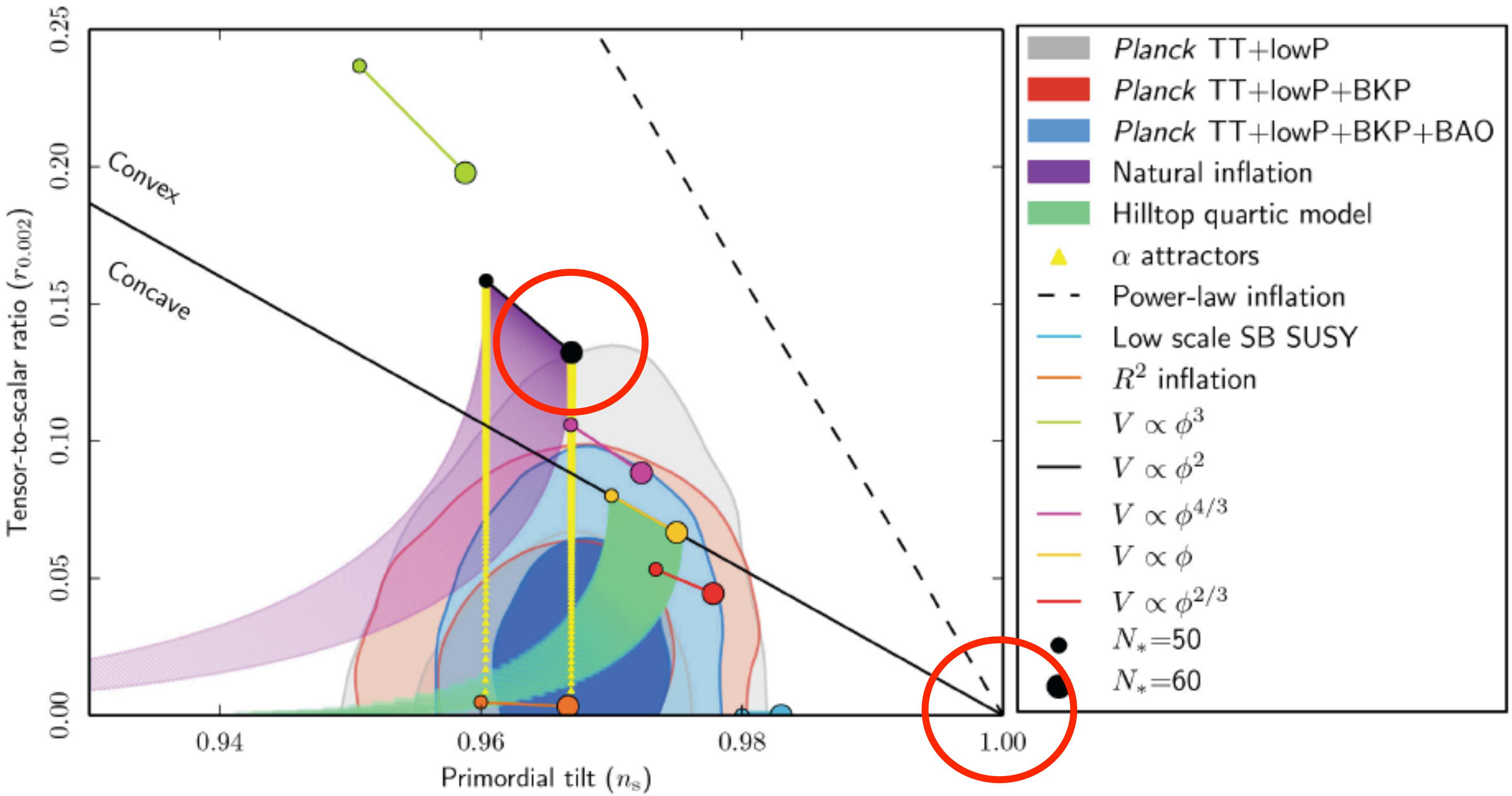
... dreams about 21cm

The plane



$$P_\zeta = A \cdot k^{-3+(n_s-1)}$$

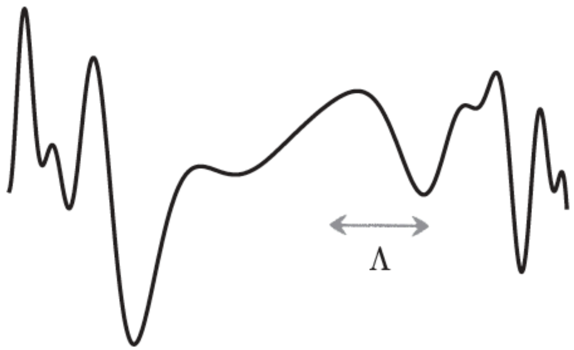
The plane



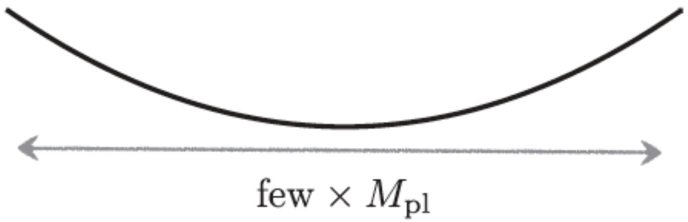
Harrison – Zeldovich and ϕ^2 gone

Mountains or hills ?

Landscape:



Around a minimum
all functions look
the same…



$$V = \frac{1}{2}m^2\phi^2$$

We now know it is not as simple as that !


GRAVITATIONAL
WAVES

The hunt for tensor modes

Present: $r < 0.07$ (BICEP-Keck 95-150 GHz + Planck)

Future. B-modes search is ongoing by many experiments:

- Ground based telescopes: ACTpol/AdvACT, CLASS, Keck/BICEP3, Qubic, Quijote, Polarbear, Simons Array, Spud, SPTpol/-3G, Stage IV;
- Balloon experiments: EBEX, Lspe, SPIDER, PIPER;
- Satellite missions: CMBPol, Pixie (NASA), EPIC (NASA), LiteBIRD (KEK), CoRE+ (ESA).

$r = 0.001$ achievable (?) even with ground-based and balloon borne experiments

(100 smaller than background)

Robust signature

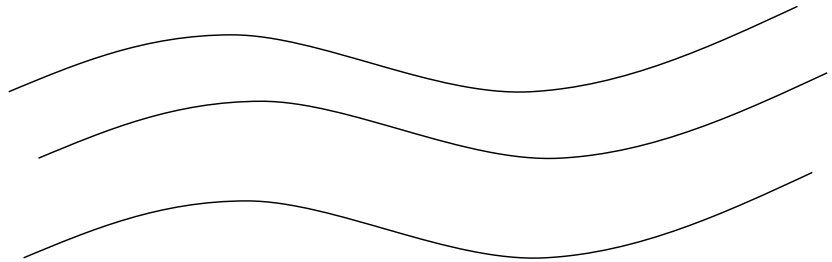
- It is easy to play with scalar perturbations:

    1. choice of potential
    2. speed of propagation c_S
    3. many scalars (effects on late Universe)

- It is not easy to play with gravity ! GWs are direct probes of H

Speed of gravity

Effective field theory of inflation:



t = const

Parametrize the most general dynamics
compatible with symmetries

$$S=\int \mathrm{d}^4x\sqrt{-g}\,\frac{M_{\mathrm{Pl}}^2}{2}\left[R-2\left(\dot{H}+3H^2\right)+2\dot{H}g^{00}-\left(1-c_T^{-2}(t)\right)\left(\delta K_{\mu\nu}\delta K^{\mu\nu}-\delta K^2\right)\right]$$

$$K_{ij}=\frac{1}{2N}\left(\dot{h}_{ij}-\nabla_i N_j-\nabla_j N_i\right)$$

$$S_{\gamma\gamma}=\frac{M_{\mathrm{Pl}}^2}{8}\int \mathrm{d}^4x\,a^3 c_T^{-2}\left[\dot{\gamma}_{ij}^2-c_T^2\frac{(\partial_k\gamma_{ij})^2}{a^2}\right] \longrightarrow \Delta_T^2=\frac{2}{\pi^2}\frac{H^2}{M_{\mathrm{Pl}}^2}\cdot\frac{1}{c_T(t)}$$

Speed of gravity

PC, Gleyzes, Noreña, Vernizzi 14

$$\Delta_T^2 = \frac{2}{\pi^2}\frac{H^2}{M_{\rm Pl}^2}\cdot\frac{1}{c_T(t)}$$

- Scale invariance without H ~ const.
- P_T does not measure energy scale
- $n_T \neq 2\dot{H}/H^2 < 0$

Speed of gravity

PC, Gleyzes, Noreña, Vernizzi 14

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PC, Gleyzes, Noreña, Vernizzi 14

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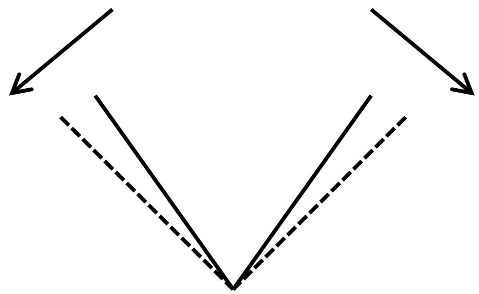
- Scale invariance without H ~ const.
- P_T does not measure energy scale
- $n_T \neq 2\dot H/H^2 < 0$

Disformal transformation:

$$\boxed{g_{\mu\nu} \mapsto g_{\mu\nu} - (1-c_T^2)\partial_\mu\phi\partial_\nu\phi/(\partial\phi)^2}$$

$$g_{\mu\nu} \mapsto c_T^{-1}(t)\, g_{\mu\nu}$$

$$\tilde t \equiv \int c_T^{1/2}(t)\mathrm{d}t\,, \qquad \tilde a(\tilde t) \equiv c_T^{-1/2}a(t)$$



$$\dot c_T = 0$$

$$\int \mathrm{d}^4x\sqrt{-\tilde g}\frac{M_{\rm Pl}^2}{2}\left\{\tilde R - 2(\dot{\tilde H}+3\tilde H^2) + 2\dot{\tilde H}\tilde g^{00} + 2(1-c_T^2)\dot{\tilde H}\times\left(1-\sqrt{-\tilde g^{00}}\right)^2\right\}$$

$$\tilde c_s = 1/c_T$$

NG in original frame beyond decoupling!

Higuchi bound

Higuchi 87

No extra light spin-2 during inflation

Spin-2 particles in de Sitter with $m^2 < 2H^2$ are forbidden (besides the graviton)

Group theoretical statement

Arkani-Hamed, Maldacena 15

$$\mathcal{O}_{ij} \sim \eta^{\Delta} \qquad \Delta_{\pm} = \frac{3}{2} \pm \sqrt{\left(s - \frac{1}{2}\right)^2 - \frac{m^2}{H^2}}$$

$$\langle \epsilon^2 . O_{\vec{k}} \tilde{\epsilon}^2 . O_{-\vec{k}} \rangle' \propto k^{2\Delta - 3} \left[e^{-2i\chi} + \frac{4(3-\Delta)}{\Delta} e^{-i\chi} + \frac{6(3-\Delta)(2-\Delta)}{(\Delta-1)\Delta} + \frac{4(3-\Delta)}{\Delta} e^{i\chi} + e^{2i\chi} \right]$$

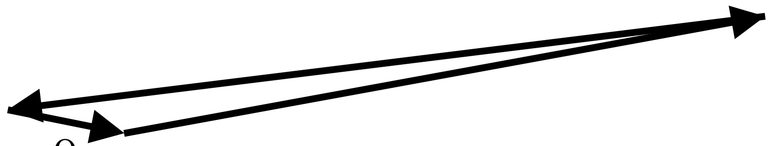
Becomes negative
for $\Delta < 1$

For example one cannot have KK gravitons with a small mass

Single-field consistency relation for 3pf

Maldacena 03
PC, Zaldarriaga 04

Squeezed limits



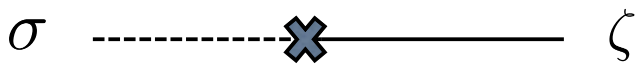
$\vec{q} \to 0$

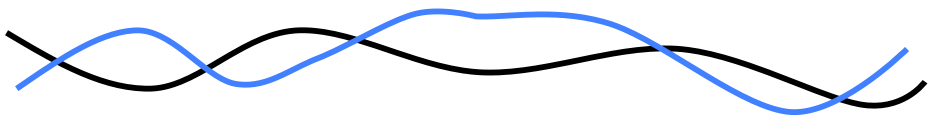
$$\phi(t, \vec{x}) = \phi_0(t) \qquad\qquad h_{ij} = e^{2\zeta(t,\vec{x})} \left(e^{\gamma(t,\vec{x})}\right)_{ij}$$

The long mode is already classical when the other freeze
and acts simply as a rescaling of the coordinates

$$\lim_{\vec{q}\to 0} \langle \zeta_{\vec{q}}\, \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle' = -\langle \zeta_{\vec{q}}\, \zeta_{-\vec{q}} \rangle' \langle \zeta_{\vec{k}_1} \zeta_{-\vec{k}_1} \rangle' \frac{\mathrm{d}\ \log k_1^3 \langle \zeta_{\vec{k}_1} \zeta_{-\vec{k}_1} \rangle'}{\mathrm{d}\log k_1}$$

Violated in multifield:





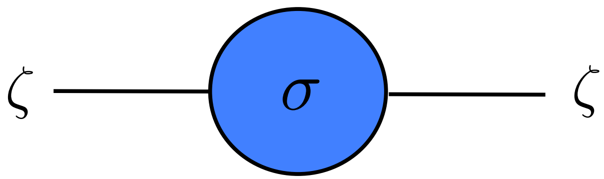
f_{NL}^{local} as smoking gun for multifield models

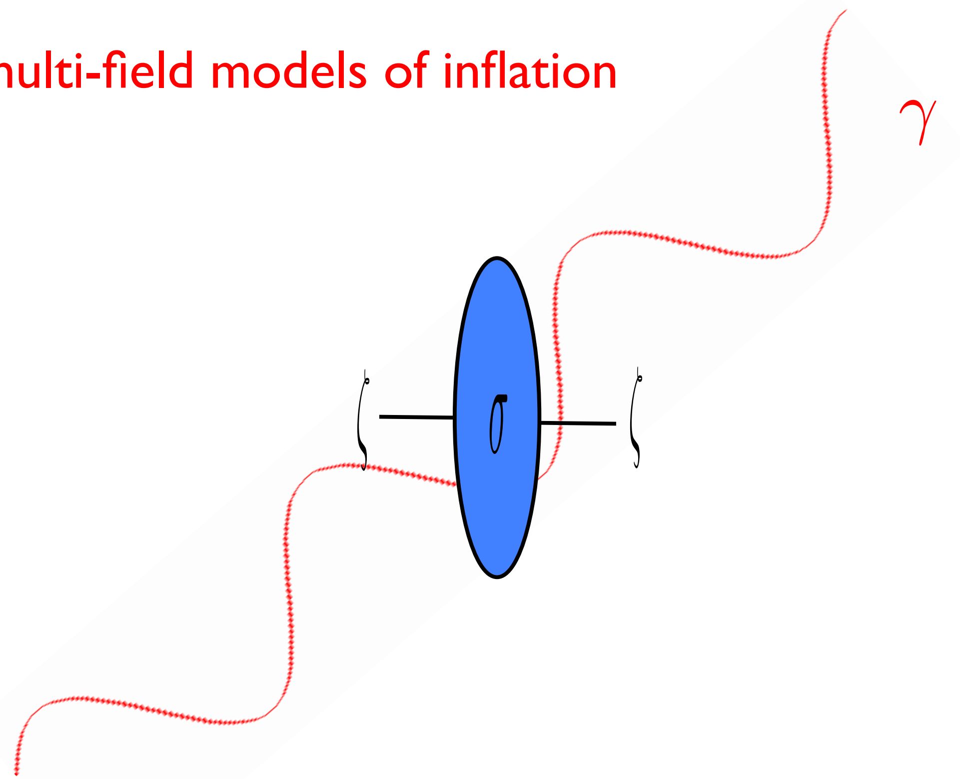
Tensor consistency relation for 3pf

Same logic leads to

$$\lim_{\vec{q}\to 0}\langle \gamma^s_{\vec{q}}\,\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\rangle' = -\langle \gamma^s_{\vec{q}}\,\gamma^s_{\vec{-q}}\rangle' \epsilon^s_{ij} k^i_1 k^j_1 \frac{\partial}{\partial k_1^2}\langle \zeta_{\vec{k}_1}\zeta_{-\vec{k}_1}\rangle'$$

But this is valid also for multi-field models of inflation





see also Dimastrogiovanni, Fasiello, Kamiokowski 15

Violated if there are extra tensors

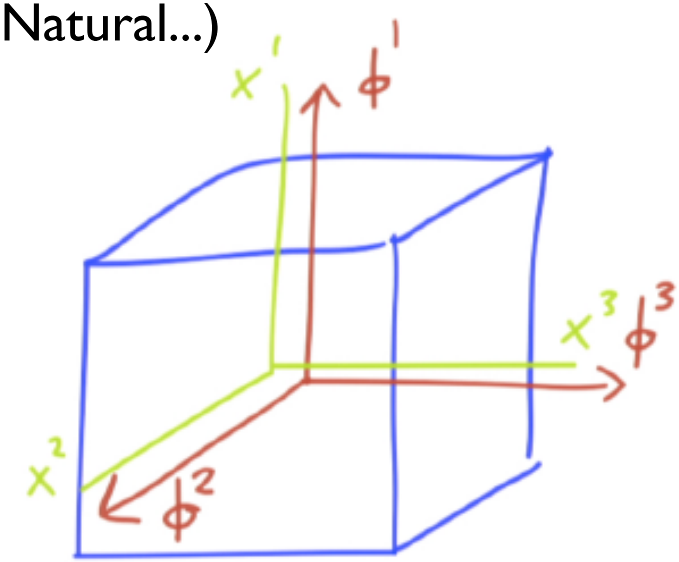
Ways out

Different symmetry patterns

Endlich, Nicolis, Wang 12

E.g. Solid inflation (also Gauge-flation, Chromo-Natural...)

CR rescaling argument fails
+ extra tensors



$$\langle \phi \rangle = c\,t \quad \to \quad \langle \phi^I \rangle = \alpha x^I$$

$$\langle \gamma^\lambda_{\vec{q}\to 0} \zeta_{\vec{k}} \zeta_{-\vec{k}} \rangle' = -\frac{10}{9}\,\frac{F_Y}{F}\, P_\gamma(q) P_\zeta(k)\, \frac{1}{c_L^2 \epsilon}\, \left(\hat{k}^i \hat{k}^j \epsilon^\lambda_{ij}\right)$$

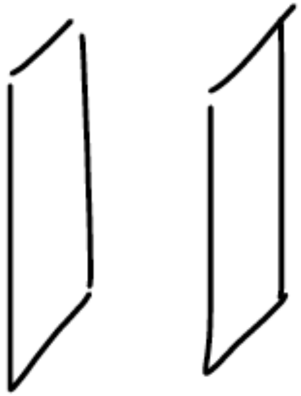
The scalar tilt

Planck: $n_s - 1 = -0.0348 \pm 0.0047 \quad (\gtrsim 7\sigma)$ It is of order 1/N (~ 0.02)

Did we expect that? Can we learn something on r?

True in many cases: $V = \frac{1}{2}m^2\phi^2$ $n_s - 1 = -\frac{2}{N}$

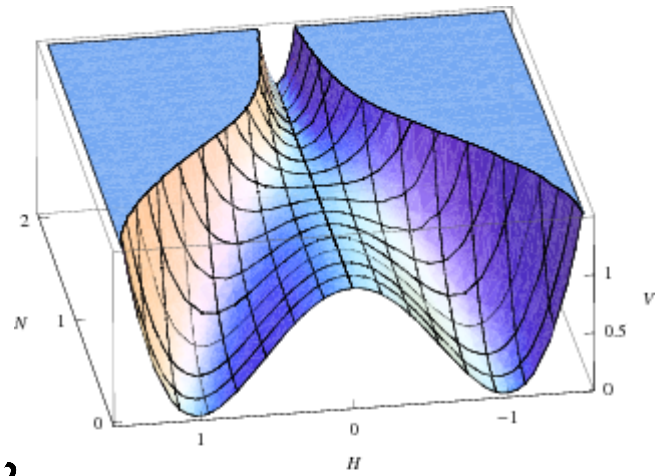
Brane inflation



$$V = V_0\left(1 - \left(\frac{\phi}{\mu}\right)^{-4}\right)$$ $$n_s - 1 = -\frac{5}{3}\cdot\frac{1}{N}$$

Starobinsky, Higgs inflation… $V \sim V_0(1 - e^{-\phi/M})$ $n_s - 1 = -\frac{2}{N}$

and not in others…



- Hybrid: $V = \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda(\psi^2 - M^2)^2 + \frac{1}{2}\psi^2\phi^2$

$n_s - 1 = 2M_P^2 m^2/V_0$ independent of N

Small but not so small because of SUGRA corrections (η-problem)?

Why not n_s -1 ~ 0.1 ?

- Natural inflation: $V = V_0\left[1 - \cos\left(\frac{\phi}{f}\right)\right]$

$$n_s - 1 = -a^2\left(1 + \frac{4}{(2+a^2)e^{a^2 N} - 2}\right) \qquad a \equiv \frac{M_P}{f}$$

It scales like 1/N only for a << 1

Let us take it seriously

PC, Dubovsky, Nacir, Simonović, Trevisan, Villadoro, Zaldarriaga 14

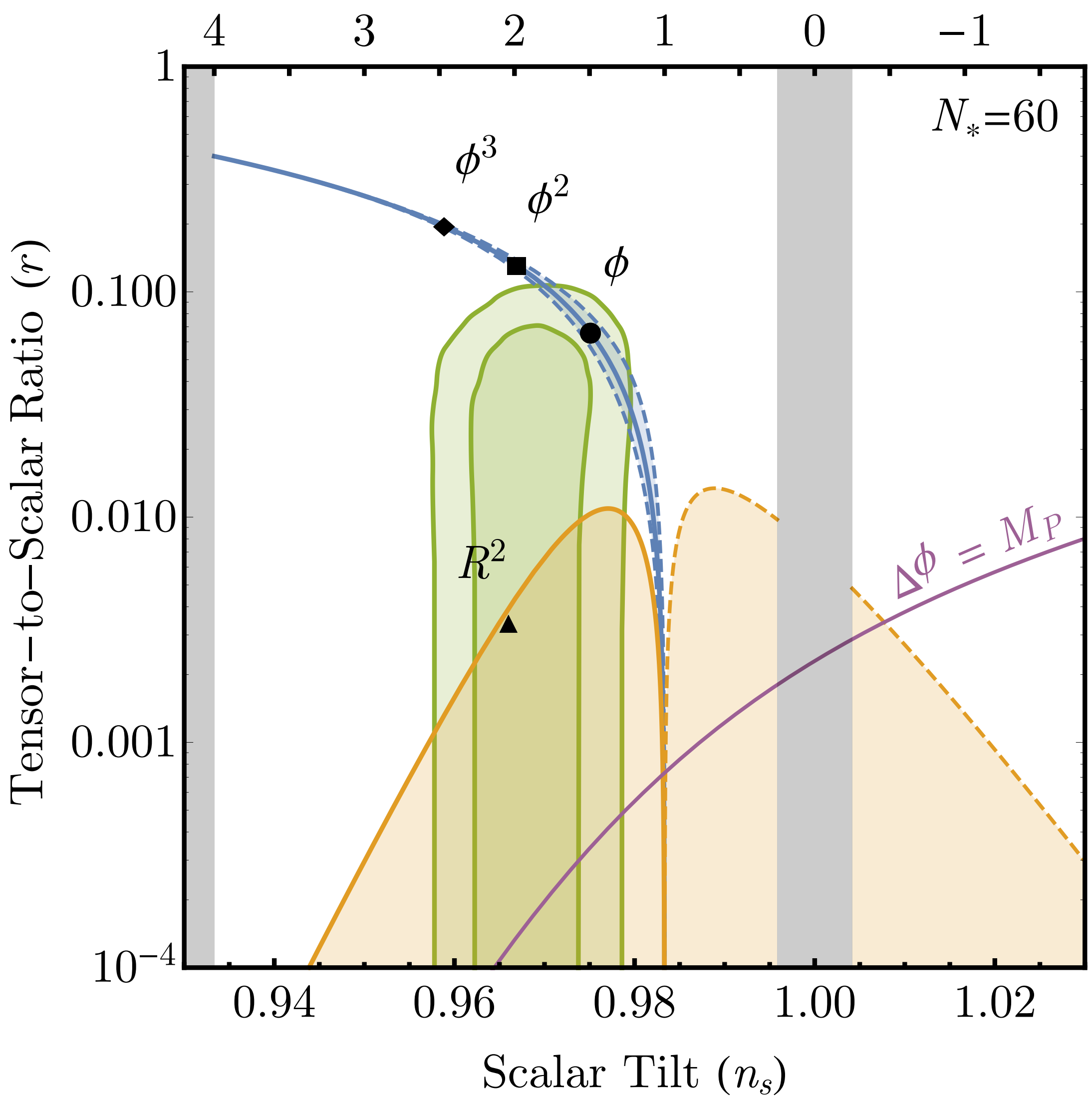
n_s -1 scales as 1/N in a window (larger than observable one)

$$n_s - 1 = -2\epsilon + \frac{\mathrm{d}\log\epsilon}{\mathrm{d}N} = -\frac{\alpha}{N}$$

$$\downarrow$$

$$\epsilon(N) = \frac{1}{2(\alpha-1)^{-1}N + AN^{\alpha}}$$

I assume one of the two scalings wins in the window



Similar to Mukhanov 13 and Roest 13

EFT of Dark Energy

PC, Luty, Nicolis, Senatore 06
Creminelli, D'Amico, Noreña, Vernizzi 07

Parametrize ignorance about dark energy (although we know it is a c.c.!)

$$S = \int d^4x \sqrt{-g} \left[\frac{M_*^2}{2} f(t)^{(4)}R - \Lambda(t) - c(t) g^{00} + \frac{M_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2}\, \delta K \delta g^{00} \right.$$
$$\left. - \; m_4^2(t) \left(\delta K^2 - \delta K^{\mu}{}_{\nu}\, \delta K^{\nu}{}_{\mu}\right) + \frac{\tilde{m}_4^2(t)}{2}\, R\, \delta g^{00} \right] \, .$$

Same logic as for inflation but:

- Coupling with matter is here relevant
- Different observables
- Usually only linear perturbation theory

Conclusions

Era of precision cosmology → Era of EFT in cosmology

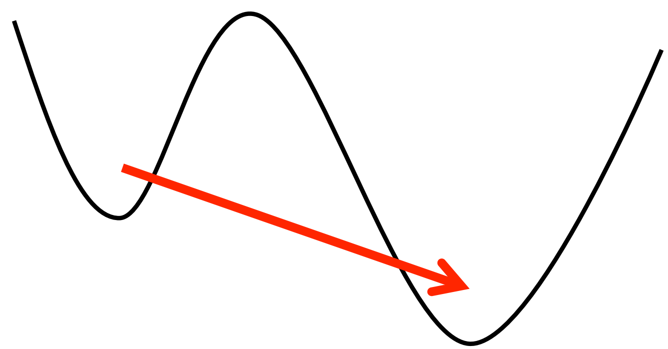
- EFT of LSS
- EFT of Bias
- EFT of Inflation
- EFT of Dark Energy

Back up slides

Advantages

- Usual approach: Start with action $L(\phi, \partial\phi, \partial\partial\phi, \ldots)$ → find inflating solution

  → expand in perturbations



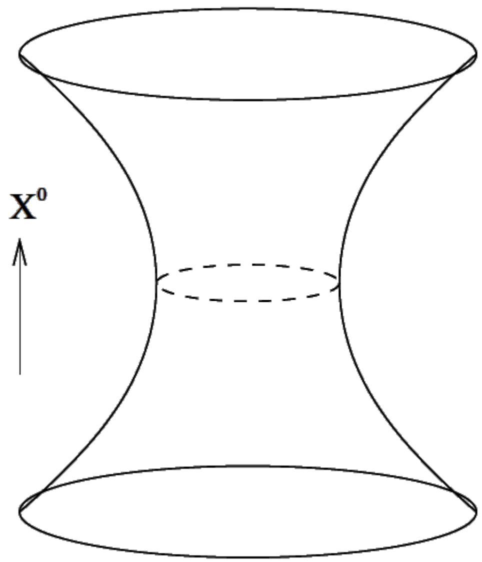
- Operators are what measured by experiments

- Not obvious the same EFT also describes very different $\dot{\phi}_0$

E.g. P(X) describes a superfluid, but only for $\dot{\phi}_0 \neq 0$

- No field redefinition ambiguity: $\phi \to \tilde{\phi}(\phi)$

de Sitter SO(4,1)



Inflation takes place in ~ dS

$$ds^2 = \frac{1}{H^2\eta^2}(-d\eta^2 + d\vec{x}^2)$$

- Translations, rotations: ok

- Dilations $\eta \to \lambda\eta, \ \vec{x} \to \lambda\vec{x}$

→ scale-invariance

$$\varphi_{\vec{k}} \to \lambda^3 \varphi_{\vec{k}/\lambda} \qquad \langle \varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\rangle = (2\pi)^3\delta(\vec{k}_1+\vec{k}_2)\frac{1}{k_1^3}F(k_1\eta)$$

assuming approximate $\phi \to \phi + c$

Disformed away

$$S = \int \mathrm{d}\tilde{t}\mathrm{d}^3x\sqrt{-\tilde{g}}\frac{M_{\rm Pl}^2}{2}\Big\{\tilde{R} - 2(\dot{\tilde{H}} + 3\tilde{H}^2) + 2\dot{\tilde{H}}\tilde{g}^{00} + \Big[2(1-c_T^2)\dot{\tilde{H}} - \quad \tfrac{3}{2}\alpha^2 - c_T^2\Big(\dot{\alpha} + \tilde{H}\alpha + \frac{1}{2}\alpha^2\Big)\Big] \times \Big(1-\sqrt{-\tilde{g}^{00}}\Big)^2 + 2\alpha\,\delta\tilde{K}\Big(1-\sqrt{-\tilde{g}^{00}}\Big)\Big\}$$

$$\alpha \equiv \dot{c_T}/c_T$$

Blue tilt using c_T → Stable NEC violation with operator $\delta N \delta K$

PC, Luty, Nicolis, Senatore 06

No loss of generality in taking $c_T = 1$
(even multifield or alternatives to inflation)

Exceptions: 1. Different symmetry pattern (solid inflation, gauge-flation…)
2. GWs not produced as vacuum fluctuations

- Running α $\quad \frac{d\epsilon^{-1}}{d\log N} - \alpha(N)\epsilon^{-1} = -2N$ $\quad \epsilon^{-1}(N) = -2e^{\int_1^N \frac{d\tilde{N}}{\tilde{N}}\alpha(\tilde{N})}\int_1^N d\tilde{N} e^{-\int_1^{\tilde{N}}\frac{d\hat{N}}{\hat{N}}\alpha(\hat{N})} + Ae^{\int_1^N \frac{d\tilde{N}}{\tilde{N}}\alpha(\tilde{N})}$.

- No lower bound on r

- "Forbidden" region: exp target

- Relevance of tilt

- Running $-\alpha/N_*^2 \simeq -7\cdot 10^{-4}$
  can we measure it ?

- c_S opens degeneracies

Zavala 14