Photon Emission in QGP using AdS/QCD models at finite chemical potential

Miguel Angel Martin Contreras

Department of Physics, University of Los Andes
Bogota, Colombia

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Outline

QGP Generalities

Generalities

- It is a deconfined state of QCD created in heavy ion collisions, where partons are almost free.
- It mimics the particle era in the early universe.
- It behaves as a perfect fluid, i.e., $\frac{S}{V} = \frac{1}{4\pi}$ and $\eta = 0$.
- It is a strongly coupled system.
- It is expected to be formed at $T_c = 175$ MeV, when light mesons melt down.
- Theoretical approaches: lattice QCD, real time methods, bootstrap models, χ-PT, Thermal Loop and other effective lagrangians.

E. Shuryak, 2004.
J. Adams et. al. (RHIC), 2005.
S. S. Adler et. al. (RHIC), 2007.

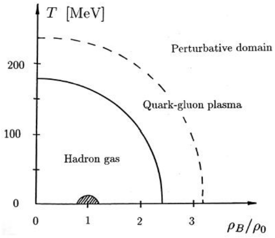

Figure 1: QCD phase diagram

Why photons?

P. Aurenche et. al., 1998, 2002.

- Photons are produced essentialy by three mechanisms:
  - Compton scattering (Pert., but suppressed by LMP effect).
  - Pair annihilation (Pert., but suppressed by LMP effect).
  - Bremsstrahlung (Non Perturb.!).
- Produced photons escaped unperturbed from the plasma ball: $\lambda_{\gamma}^{\text{Mean Free Path}} > R_{\text{Plasma ball}}$.
- Photons carry valuable information about topological properties of the colored medium at such temperatures.

Thus, we conclude that photons emitted in the QGP scenario are good probes to study QGP properties.

Photon Emission Rate

P. Arnold, et. al., 2001.

Consider a QGP defined by a colored quantum system in equilibrium, described by a thermal field theory at the strongly coupled limit. Assume also that electromagnetic interaction between matter and photons is given by the electromagnetic current, $e\,A^{\mu}\,J_{\mu}^{em}$, thus at leading order in e we have

$$d\Gamma_{\gamma} = \frac{d^3q}{(2\pi)^3}\,\frac{e^2}{2\left(k^0\right)^2}\,\eta^{\mu\nu}\,C_{\mu\nu}^{R}\left(k\right)\Big|_{k^0=|\vec{q}|}, \qquad (1)$$

where we have defined the photon 4-momentum as $k^{\mu} = \left(k^0, \vec{q}\right)$, $\eta_{\mu\nu}$ the Minkowski metric and $C_{\mu\nu}^{R}\left(k\right)$ is the Wightman function defined for electromagnetic currents as,

$$C_{\mu\nu}^{R}\left(k\right) = \int d^4x\, e^{-ik\cdot x}\,\langle J_{\mu}^{EM}\left(0\right)\; J_{\nu}^{EM}\left(x\right)\rangle. \qquad (2)$$

The expectation value in (2) is taken in the thermal equilibrium state and $x^{\mu} = \left(x^0, \vec{x}\right)$.

In the thermal equilibrium limit the Wightman correlator can be written in terms of the spectral density,

$$\begin{aligned} C^R_{\mu\nu} &= n_B\left(k^0\right)\,\chi_{\mu\nu}\left(k\right), && (3)\\ \chi_{\mu\nu} &= -2\,\mathbb{I}\mathrm{m}\,G^R_{\mu\nu}\left(k\right), && (4) \end{aligned}$$

with n_B the Bose-Einstein distribution and G^R the thermal retarded Green function. Since the idea is to model real photons (implying $k^0 = |\vec{q}|$), it is possible to ignore the longitudinal part and focus on the transversal one only. Then, the trace of the spectral function (4) is written as

$$\chi^{\mu}_{\mu}\left(k\right) = -4\,\mathbb{I}\mathrm{m}\,\Pi^T\left(k\right), \tag{5}$$

where Π^T is the transversal part of the spectral density.Therefore, the photon emission rate is given by

$$\frac{d\Gamma_\gamma}{d^3|\vec{q}|} = \frac{e^2}{\left(2\pi\right)^3\,2|\vec{q}|}\,n_B\left(k^0\right)\,\chi^{\mu}_{\mu}\left(k\right)\Bigg|_{k^0=|\vec{q}|}\,. \tag{6}$$

Conclusion

Thus, the important object to be calculated is the trace of the spectral density $\chi_{\mu}^{\mu}(k)$*.*

Other interesting observables coming from the spectral function

The electrical AC and DC conductivities of the medium are also determined by $\chi_{\mu\nu}$. From the Kubo formula the AC $\sigma\left(k^0\right)$ conductivity can be read from the spatial components as

$$\sigma_{\text{AC}}\left(k^0\right) = -\frac{\chi_{ii}\left(k^0, \vec{k} = 0\right)}{2ik^0}. \tag{7}$$

From the trace of the spectral density in the limit of $k_0 \to 0$ we can read the DC conductivity as

$$\sigma_{\text{DC}} = \frac{\alpha_{\text{em}}\pi}{2T}\frac{d\chi_{\mu}^{\mu}\left(k^0\right)}{dk^0}\bigg|_{k^0=0}. \tag{8}$$

Let's compute these objects using holographic methods!

AdS/CFT Correspondence

A possible definition...

A strongly coupled QFT living in $d+1$ dimensions (boundary) is equivalent to a weakly coupled gravity theory living in $d+2$ dimensions (bulk).

Implications

- Space–time data encoded into QFT (V. Hubbeny).
- Saddle point approx.: Classical Gravity can be used to explore non-pertubative QFT. (MAGOO, 1999).
- Every field ϕ in the bulk is a *Schwinger source* of an operator $\mathcal{O}$ at the boundary.
- Bulk physics is equivalent to boundary physics.

Summarizing:

$$e^{W[\phi]}\Big|_{\text{Boundary}} = \langle e^{\int \phi\, \mathcal{O}} \rangle\Big|_{\text{QFT}} \tag{9}$$

With $W[\phi]$ the functional generator for the n-point functions of $\mathcal{O}$:

$$\langle \mathcal{O} \ldots \mathcal{O} \rangle = \left. \frac{\delta W}{\delta \phi} \right|_{\phi=0,\text{ evaluated at the boundary}} \tag{10}$$

Holographic Algorithm

- Define a gravitational action for the bulk physics.
- Solve the equations of motion and obtain the on–shell boundary action.
- Use (10) to obtain the n-point function.
- Find the map between the observables in the QFT and the bulk quantities (i.e. the holographic dictionary).

In our case QGP a is finite temperture system. Thus, the (thermal) field theory associated to QGP is dual to a black hole solution in AdS. Confinement/deconfiment phase transition is given by a Hawking Page transition (E. Witten, 1998).

Holographic Photon Emission

A summary of holographic approaches

- At zero chemical potential.
  - PER in Dp/Dq systems: Mateos et. al. (2009).
  - PER in AdS/QCD soft wall model models: A. Nata Atmaja and K. Schalm, 2010.
  - PER in anysotropic plasmas (Dp/Dq system): D. Trancanelli and L. Patiño, (2010).
- At finte chemical potential:
  - PER in AdS/QCD models and Sakai–Sugimoto (introducing chem. pot. using DBI action fluctiations): Y. Y. Bu, 2012.

Another possibility to model finite chemical potential is using electrically charged black hole solutions in AdS (S. Hawking et. al. 1999 and R. G. Cai et. al. 2007).
Therefore, in the next slides we will calculate PER at finite chem. pot. following a different path: using AdS–Reissner–Nordstrom solution.

AdS–RN background

AdS–RN BH is a solution of

$$I_{\text{Bulk}} = \frac{1}{16\,\pi\,G_5}\int d^5x\,\sqrt{-g}\,e^{-\Phi}\left(\mathcal{R} - 2\Lambda - \frac{1}{4g_5^2}G_{mn}G^{mn}\right), \quad (11)$$

where $G_{mn} := \partial_m\,V_n\,(z,x^\mu) - \partial_n\,V_m\,(z,x^\mu)$, the cosmological constant is $\Lambda = -\frac{6}{R^2}$, with R the AdS radius and $\frac{1}{g_5^2} = \frac{N_c\,N_f}{4\pi}$. The dilaton Φ characterizes the type of AdS/QCD model in case. The solution of (11) is written as

$$dS^2 = \frac{L^2}{R^2}\left[-h\,(z)\,dt^2 + d\vec{x}^2 + \frac{dz^2}{h\,(z)}\right]. \quad (12)$$

The blackening factor $h\,(z)$ is fixed by the 1-form V_m. If a static solution is chosen, $V_m = -V\,(z)\;dt$, the h factor can be written as

$$h\,(z) = 1 - (1 + q^2 z_h^6)\left(\frac{z}{z_h}\right)^4 + q^2 z_h^6\left(\frac{z}{z_h}\right)^6, \quad (13)$$

where q is the charge of the BH and z_h is the minimal event horizon.

Temperature is defined using the Hawking–Page relation (S. Hawking et. al. 1999 and R. G. Cai et. al. 2007)

$$T = \frac{|h'(z_h)|}{4\pi} = \frac{1}{\pi\, z_h}\left(1 - \frac{q^2\, z_h^6}{2}\right). \qquad (14)$$

For the chemical potential μ, first we need to obtain, by solving the potential $V(z)$ from the e.o.m and imposing that $V(z \to 0) = \mu$ and $V(z_h) = 0$, the on–shell bulk action $I_{\text{Bulk}}^{\text{On-shell}}$. Second, we construct the grand canonical potential as

$$\Omega(\mu, T) = I_{\text{Bulk}}^{\text{On–shell}} \qquad (15)$$

The chemical potential is studied through the (holographic baryonic) U(1) charge q (S. Nakamura, 2007 and 2008). To do so, we take the derivative of $\Omega(\mu, T)$ in terms of μ:

$$q = -\left.\frac{\partial \Omega}{\partial \mu}\right|_T . \qquad (16)$$

Notice that this procedure is depending on the form of the dilaton. AdS/QCD models are generated supposing static profiles for Φ. Let's focus on this now!

Holographic model for photons

Photons created in the QGP will be modeled using a 5-dimensional $U(1)$ field A_m coupled to Φ:

$$I_\gamma = -\frac{1}{4g_\gamma^2}\int d^4x\,\sqrt{-g}\,e^{-\Phi(z)}F_{mn}F^{mn}, \tag{17}$$

where g_γ^2 is the coupling constant for photons in the QGP, that in general depends on temperature (D. Mateos et. al. 2007). For these static dilaton models, this coupling is defined as $1/g_\gamma^2 = N_c N_f T^2/8\pi^2 \equiv \mathcal{N}$.
From the action (17) we can obtain the e.o.m. for the photon field

$$\partial_z\left[\frac{e^{-\Phi(z)}}{z}h(z)\,\partial_z A(q,z)\right] + \frac{e^{-\Phi(z)}}{z}\,k_0^2\,\left[1-h(z)\right]A(q,z) = 0, \tag{18}$$

where the gauge fixing $A_z = 0$ was implicitly used and $A(q,z)$ is the transversal field component in Fourier space. The longitudinal components, related to the dilepton production are fixed to be zero.

Holographic spectral function

χ^{μ}_{μ} is obtained via the retarded thermal Green function G^R, from the on–shell boundary photon action

$$I^{\text{Bndry}}_{\text{On–Shell}} = -\frac{L}{2\,g_\gamma^2}\int d^4q\,\left[\frac{e^{-\Phi(z)}}{z}\,h(z)\,A(z,q)\,\partial_z A(z,-q)\right]^{z=z_h}_{z\to 0}. \tag{19}$$

Following the *minkowskian prescription* (Starinets et. al. 2002), the thermal Green function is given by

$$G^R(q) = \frac{1}{g_\gamma^2}\,\frac{e^{-\Phi(z)}}{z}\,h(z)\,A(z,q)\,\partial_z A(z,-q)\bigg|_{z=z_h}. \tag{20}$$

Therefore, the trace of the spectral function is

$$\chi^{\mu}_{\mu}(q) = -\,4\,\frac{e^{-\Phi(z)}}{z}\,h(z)\;\mathbb{Im}\,A(z,q)\,\partial_z A^*(z,q)\bigg|_{z=z_h}. \tag{21}$$

Hard wall model results

- Introduce a hard cutoff $z_{\Lambda_{\text{QCD}}}$ in the AdS-RN space, such that $z_{\Lambda_{\text{QCD}}} > z_h$ (C. Herzog, 2005).
- Fix the dilaton profile as $\Phi = 0$.

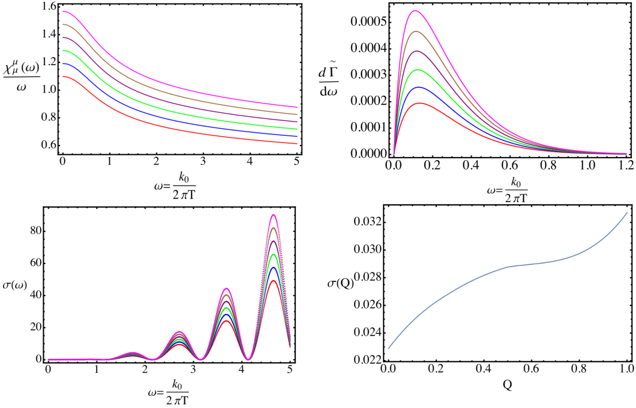

Figure 2: Upper panels: χ^{μ}_{μ} and the PEM normalized by $\mathcal{N} = N_c N_f T^2/8\pi^2$ as a function of Q from 0 (red) up to 1.0 (magenta). Lower panels: AC (left) and DC (right) conductivities.

Soft Wall Model Results

- Define a soft cutoff by fixing the dilaton profile as $\Phi(z) = \kappa^2 z^2$.
- κ induces confinement and it is defined in terms of temperature and the baryonic charge $Q = q z_h^3$ (C. Herzog et al, 2005; R.G Cai et. al., 2007 and S. Nakamura, 2007).

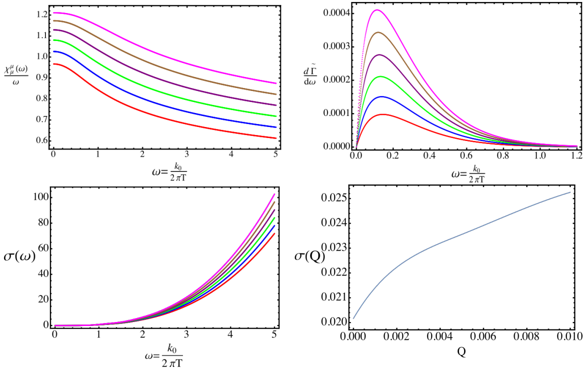

Figure 3: Upper panels: χ^μ_μ and the PEM nomalized by $\mathcal{N}$ for Q running from 0 (red) up to 0.01 (magenta). Lower panels: AC (left) and DC (right) conductivities.

Conclusions

- We have calculated the photon emission rate for AdS/QCD hard wall and soft wall models in the finite chemical potential regime modeled by an AdS–RN background.
- We have calculated the AC and DC conductivities for both AdS/QCD models.
- In the perspective of QFT, the holographic procedure showed here is equivalent to consider all the possible Feynman diagrams allowed by the Landau–Pomeranchuk–Midgal effect with photons as external legs.

Future work

- To extend these ideas to other holographic bottom/up models, as the one recently developed by N. R. F Braga, *M. A. Martin Contreras* and S. Diles (EPJ C 76(11):598, 2016) or the dynamical soft wall model (B. Batell and T. Gherghetta, 2008).
- To work these approximations in the top/down AdS bootstrap extension (M. F. Paulos et. al. 2016).
- To Study other QGP properties in these AdS/QCD models at finite chemical potential, such as the jet quenching parameter.
- To explore the low temperature and high baryonic charge limit, in order to study holographic nuclear matter and holographic neutron stars.

Thank you!