Pomeron and Odderon Regge trajectories from AdS/QCD models

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in "XI SILAFAE – Latin American Symposium on High Energy Physics "

Antigua Guatemala, Guatemala, November 14th to 18th, 2016

Work done in collaboration with:
Henrique Boschi-Filho and Danning LI

Summary of the talk:

- **Brief Review: AdS/CFT correspondence and AdS/QCD**

- **The Hard wall model - Glueballs in AdS/QCD: the Odderon**

- **The Soft wall model and some variations - Glueballs in AdS/QCD: the Pomeron and the Odderon**

AdS/CFT correspondence, J. Maldacena, 1997

(simplified version of a particular useful case)

SUPERSTRING THEORY in the AdS_5 x S^5 spacetime.

DUAL

YANG-MILLS THEORY

- Supersymmetric $\mathcal{N} = 4$
- Conformal
- SU(N) symmetry, with $N \to \infty$

in a 4-dimensional Minkowski spacetime (AdS_5x S^5 boundary).

At low energies string theory is represented by an effective supergravity theory → **gauge / gravity duality**

Other versions of the Correspondence: AdS_4 x S^7 or AdS_7 x S^4 (M-theory in 11 dimensions)

- After breaking the conformal symmetry one can build phenomenological models that describe approximately QCD. So, AdS/QCD models.
- Strong coupling theory ⇔ Weak coupling theory.

Quantum Chromodynamics - QCD

✓ used as the standard theory to explain the phenomenology of strong interactions.

❑ at the low-energy limit ($g_{YM} > 1$) the QCD cannot be treated perturbatively.

❖ Regge trajectories are an example of nonpertubative behavior of strong interactions: difficult to model it using QCD.

Alternative Way

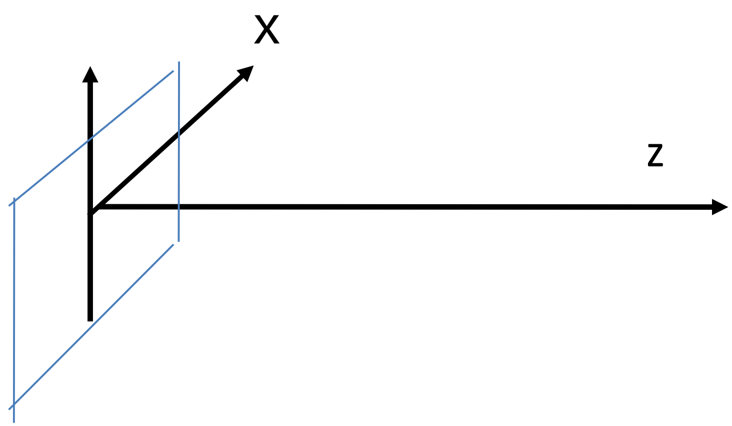
AdS/CFT correspondence

Holography in String theory

AdS Space in Poincaré coordinates

$$ds^2 = \frac{R^2}{(z)^2}(dz^2 + (d\vec{x})^2 - dt^2)$$

The 4-dim boundary is at z = 0



Fifth dimension z ~ 1 / E where E = Energy in 4-dim boundary

Hard-wall Model

Polchinski & Strassler 2001/2002

Scattering of Glueballs using the AdS/CFT correspondence

Finite region in AdS space $0 < z < z_{max}$

$z_{max} \sim 1 / E$ where E is the Energy scale in boundary theory

HBF & Braga JHEP 2003, EPJC 2004

Masses of Glueball states 0++ and its radial excited states 0++*, 0++**, 0++***, ...

Brodsky, Teramond PRL 2005, 2006; Erlich, Katz, Son, Stephanov PRL 2005.

Extension to Mesons and Baryons

Regge Trajectories

Strongly interacting particles (Hadrons) obey approximate relations between Angular Momentum (J) and quadratic masses (m^2)

$$J(m^2) \approx \alpha_0 + \alpha' m^2$$

Where α_0 and α' are constants

Extended for glueball: J^{PC}

$$\mathcal{L}_{QCD} = \bar{\psi}\left(i\not{D} - m\right)\psi - \frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu},$$

The Pomeron

Experimental Regge trajectories from proton proton scattering

$$J(m^2) \approx 1.08 + 0.25m^2$$

Masses *m* in GeV (A. Donnachie and P. V. Landshof, Nucl. Phys. B 267, 690 (1986))

The Pomeron is related to Glueball states $\mathbf{2^{++}, 4^{++}, 6^{++}, 8^{++}}$

and may be to $\mathbf{0^{++}}$

Odderon Regge trajectories

Llanes-Estrada, Bicudo and Cotanch, PRL 2006

Relativistic many-body model (RMB)

$$J(m^2) \approx -0.88 + 0.23m^2$$

Non-relativistic constituent model (NRCM)

$$J(m^2) \approx 0.25 + 0.18m^2$$

The Odderon is related to Glueball states $\mathbf{3^{--}, 5^{--}, 7^{--}, 9^{--}}$

and may be to $\mathbf{1^{--}}$

Experimental signs of the Odderon

The best experimental evidence for the odderon occurred in 1985 at ISR CERN. A difference between differential cross sections for pp and $p\bar{p}$ in the dip-shoulder region $1.1 < |t| < 1.5$ GeV^2 at $\sqrt{s} = 52.8$ GeV was measured, but these results were not confirmed [14].

There are two more evidences related to the nonperturbative odderon, that is, the change of shape in the polarization in $\pi^- p \to \pi^0 n$ from $p_L = 5$ GeV$/c$ [16,17] to $p_L = 40$ GeV$/c$ [18] and a strange structure seen in the $UA4/2$ dN/dt data for pp scattering at $\sqrt{s} = 541$ GeV, namely a bump centered at $|t| = 2 \times 10^{-3}$ GeV^2 [19].

[14] R. Avila, P. Gauronm, and B. Nicolescu, Eur. Phys. J. C **49**, 581 (2007).

[15] Z.-H. Hu, L.-J. Zhou, and W.-X. Ma, Commun. Theor. Phys. **49**, 729 (2008).

[16] D. Hill *et al.*, Phys. Rev. Lett. **30**, 239 (1973).

[17] P. Bonamy *et al.*, Nucl. Phys. **B52**, 392 (1973).

[18] V. D. Apokin *et al.*, AIP Conf. Proc. **95**, 118 (2008).

[19] C. Augier *et al.* (UA4/2 Collaboration), Phys. Lett. B **316**, 448 (1993).

LCH new results? Some groups are looking for the Odderon...

Odd spin (*P=C=-1*) Glueballs and the Odderon

EFC and H. Boschi PRD 2013

Massive scalar fields in AdS^5

$$\left[z^3\partial_z \frac{1}{z^3}\partial_z + \eta^{\alpha\beta}\partial_\alpha\partial_\beta - \frac{m_5^2R^2}{z^2}\right]\phi(x,z) = 0,$$

$$m_5^2R^2 = (\Delta - p)(\Delta + p - 4). \quad (p=0)$$

$$\phi(x,z) = A_{\nu,k}\exp^{-ip.x} z^2 J_\nu(u_{\nu,k}z),$$

$$\nu = \sqrt{4 + m_5^2R^2},$$

Boundary operator

$$\mathcal{O}_6 = SymTr\left(\tilde{F}_{\mu\nu}F^2\right),$$

conformal dimension $\Delta = 6.$

Insertion of symmetrized covariant derivatives

$$\mathcal{O}_{6+J} = SymTr\left(\tilde{F}_{\mu\nu}F D_{\{\mu 1 \ldots} D_{\mu J\}} F\right)$$

$$\Delta = 6 + \ell$$

spin $\ell = J \geq 1 \Longrightarrow \nu = 4 + \ell$

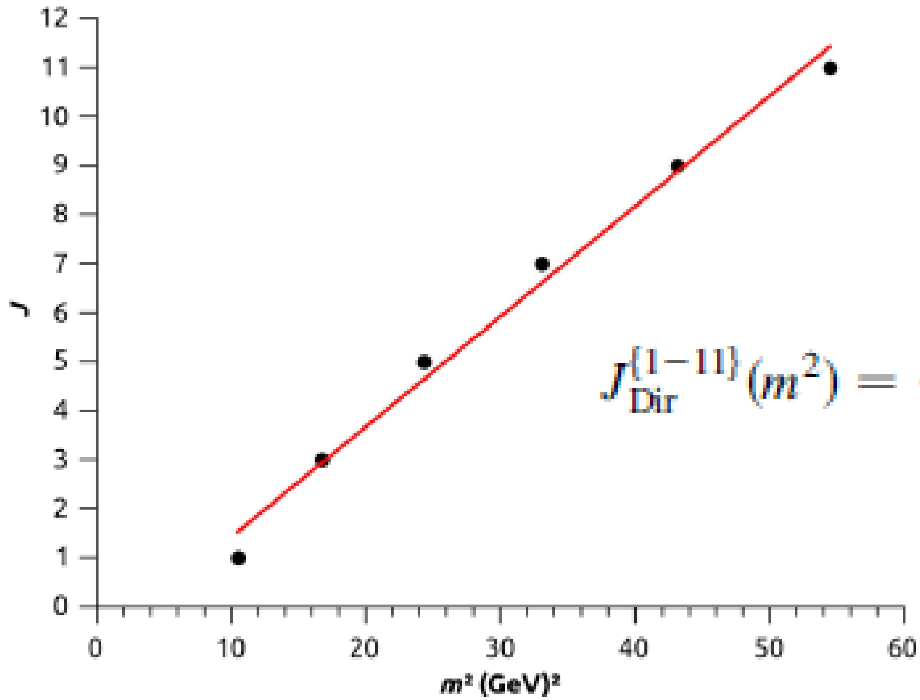
glueball states $1^{--}, 3^{--}, 5^{--}$, etc.

TABLE I. Glueball masses for states J^{PC} expressed in GeV, with odd J estimated using the hardwall model with Dirichlet and Neumann boundary conditions. The mass of 1^{--} is used as an input from the isotropic lattice [36,37]. We also show other results from the literature for comparison.

	Glueball states J^{PC}					
Models used	1^{--}	3^{--}	5^{--}	7^{--}	9^{--}	11^{--}
Hardwall with Dirichlet b.c.	3.24	4.09	4.93	5.75	6.57	7.38
Hardwall with Neumann b.c.	3.24	4.21	5.17	6.13	7.09	8.04
Relativistic many body [1]	3.95	4.15	5.05	5.90		
Nonrelativistic constituent [1]	3.49	3.92	5.15	6.14		
Wilson loop [38]	3.49	4.03				
Vacuum correlator [39]	3.02	3.49	4.18	4.96		
Vacuum correlator [39]	3.32	3.83	4.59	5.25		
Semirelativistic potential [40]	3.99	4.16	5.26			
Anisotropic lattice [41]	3.83	4.20				
Isotropic lattice [36,37]	3.24	4.33				

[36] H. B. Meyer and M. J. Teper, Phys. Lett. B 605, 344 (2005).
[37] H. B. Meyer, arXiv:hep-lat/0508002.

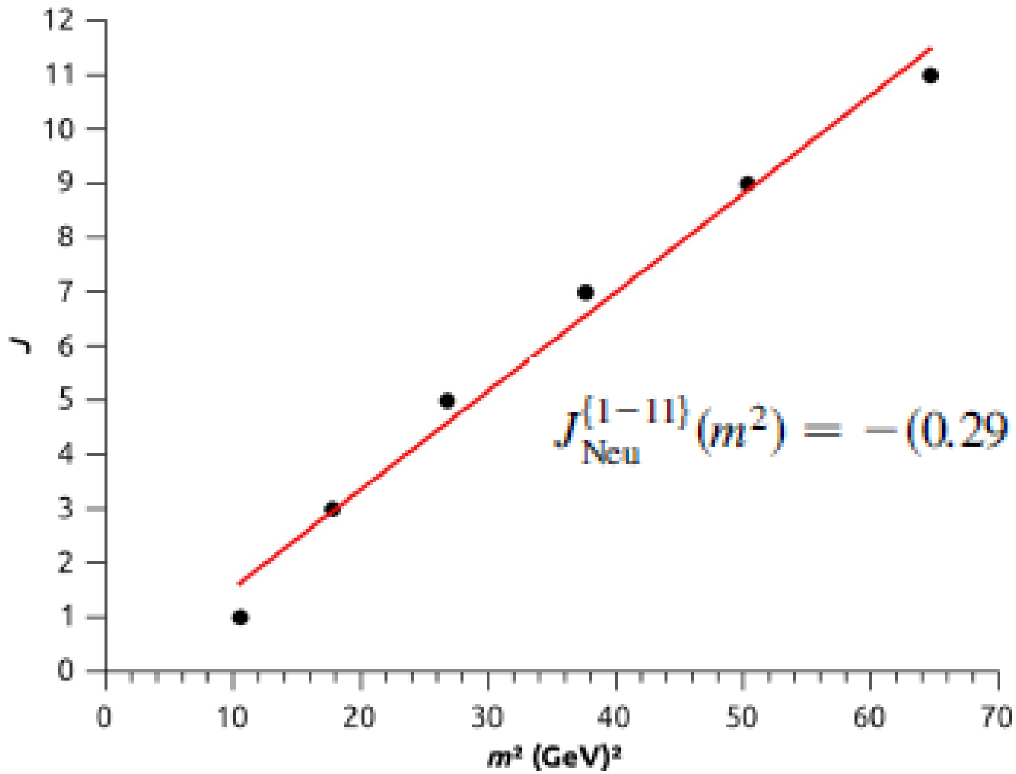
Odd Glueball states in the Hard-wall with Dirichlet Boundary condition



$$J_{\rm Dir}^{\{1-11\}}(m^2) = -(0.83 \pm 0.40) + (0.22 \pm 0.01)m^2.$$

Good agreement with the Relativistic Many-body Model (RMB)

Odd Glueball states in the Hard-wall with Neumann Boundary condition



$$J_{\rm Neu}^{\{1-11\}}(m^2) = -(0.29 \pm 0.42) + (0.18 \pm 0.01)m^2.$$

Good agreement with Non-relativistic constituent model (NRCM)

The complete work can be seen in Phys.Rev. D88 (2013) no.2, 026010 [arXiv:1301.4457 [hep-th]]

Open questions for the Odderon

Experimental confirmation?

The authors

F. J. Llanes-Estrada, P. Bicudo, and S. R. Cotanch, Phys. Rev. Lett. 96, 081601 (2006).

suggest that the state 1^{--} does NOT belong to the Odderon trajectory

Our analysis with the Hard-wall is not conclusive in this regard

Soft-wall AdS/QCD Model (T=0)

Soft cut off (Karch, Katz, Son, Stephanov PRD 2006)

$$\int d^5x \sqrt{-g}\,\mathcal{L} \quad \Longrightarrow \quad \int d^5x \sqrt{-g}e^{-\Phi}\mathcal{L}\,. \quad ; \quad \Phi(z) = cz^2$$

spectrum of vector mesons $\qquad m^2_{V_n} = 4c(n+1)\,,$

Glueballs in the soft-wall (T=0)
[Colangelo, De Fazio, Jugeau, Nicotri PLB(2007)]

The corresponding glueball spectrum is

$$m^2_{G_n} = 4c(n+2)\,.$$

Softwall Model

1

Colangelo et al 2007 (scalar, vector and tensor glueballs)
EFC and Henrique Boschi-Filho 2016 (higher spin glueballs)

$$S = \frac{\pi^3 R^5}{4\kappa^2} \int d^5x \sqrt{-g} e^{-\phi(z)} \left[g^{mn} \partial_m \mathcal{G} \partial_n \mathcal{G} + M_5^2 \mathcal{G}^2 \right]$$

$$ds^2 = \frac{R^2}{z^2}(-dt^2 + d\vec{x}^2 + dz^2) \Leftrightarrow e^{A_s(z)}(-dt^2 + d\vec{x}^2 + dz^2); \; A_s(z) = \ln(\frac{R^2}{z^2})$$

$\phi = kz^2$ and $k \sim \Lambda_{QCD}^2$

$$\partial_m [\sqrt{-g}\, e^{-\phi(z)} g^{mn} \partial_n \mathcal{G}] - \sqrt{-g} e^{-\phi(z)} M_5^2 \mathcal{G} = 0$$

Softwall Model

2

$$-\psi''(z) + \left[k^2 z^2 + \frac{15}{4z^2} + 2k + \left(\frac{R}{z}\right)^2 M_5^2\right]\psi(z) = (-q^2)\psi(z)$$

which has a well known solution:

$$\psi_n(z) = \mathcal{N}_n\, z^{t(M_5)+\frac{1}{2}}\, {}_1F_1(-n; t(M_5)+1, kz^2)\exp\{-kz^2/2\}$$

The corresponding "eigenenergies" $-q^2 = -q_\mu q^\mu$ are identified with the 4-d glueball squared masses

$$m_n^2 = \left[4n + 4 + 2\sqrt{4 + M_5^2 R^2}\right]k; \quad (n = 0, 1, 2, \cdots).$$

from AdS/CFT dictionary ⇒

$$\Delta = 2 + \sqrt{4 + R^2 M_5^2}$$

Higher spin in the Softwall Model

1

Analoglous to the higher spin Glueballs in the Hardwall

Even spin

$$\mathcal{O}_4 = Tr(F^2) = Tr(F^{\mu\nu} F_{\mu\nu})$$

$$\mathcal{O}_{4+J} = F D_{\{\mu 1 \ldots} D_{\mu J\}} F \,, \Rightarrow \quad \Delta = 4 + J$$

$$m_n^2 = \left[4 + 2\sqrt{4 + J(J+4)}\right] k \,; \qquad (\text{even } J).$$

Odd spin

$$\mathcal{O}_6 = SymTr\left(\tilde{F}_{\mu\nu} F^2\right)$$

$$\mathcal{O}_{6+J} = SymTr\left(\tilde{F}_{\mu\nu} F D_{\{\mu 1 \ldots} D_{\mu J\}} F\right) \Rightarrow \quad \Delta = 6 + J$$

$$m_n^2 = \left[4 + 2\sqrt{4 + (J+6)(J+2)}\right] k; \quad (\text{odd } J).$$

Results are not good when compared with the literature!!!

PLB (2016) [arXiv:1510.03372 [hep-ph]]

$\mathbf{0^{++}, 0^{++*}, 0^{++**}, 0^{++***}}$

$\mathbf{2^{++}, 4^{++}, 6^{++}, 8^{++}}$

$\mathbf{1^{--}, 3^{--}, 5^{--}, 7^{--}, 9^{--}}$

Regge trajectories for the pomeron and the odderon.

A Dynamical Softwall Model 1 (DSW)

D. Li & Huang JHEP 2013 (scalar glueballs)
EFC, D.Li, HBF 2016 (higher spins)

This dynamical model that takes into account dynamical corrections to the metric of the anti de Sitter space.

Now the warp factor is given by:

$$A_s(z) = \log\left(\frac{R}{z}\right) + \frac{2}{3}\Phi(z) - \log\left({}_0F_1\left(\frac{5}{4}, \frac{\Phi^2}{9}\right)\right)$$

$\phi = kz^2$ and $k \sim \Lambda^2_{QCD}$

which implies that dynamical model is no longer AdS, but it is asymptotically AdS in the limit $z \to 0$.

$$-\psi''(z) + \left[k^2z^2 + \frac{15}{4z^2} - 2k + \left(\frac{RM_5}{z}\right)^2 e^{4kz^2/3}\mathcal{A}^{-2}\right] \times\psi(z) = -q^2\psi(z).$$

$\mathcal{A}$ is given by ${}_0F_1(\frac{5}{4}, \frac{\Phi^2}{9})$

A Dynamical Softwall Model 2

➤ REMEMBER

$$A_s(z) = \log\left(\frac{R}{z}\right) + \frac{2}{3}\Phi(z) - \log\left({}_0F_1\left(\frac{5}{4}, \frac{\Phi^2}{9}\right)\right)$$

$$-\psi''(z) + \left[k^2 z^2 + \frac{15}{4z^2} - 2k + \left(\frac{RM_5}{z}\right)^2 e^{4kz^2/3}\mathcal{A}^{-2}\right] \times \psi(z) = -q^2\psi(z).$$

$\mathcal{A}$ is given by ${}_0F_1(\frac{5}{4}, \frac{\Phi^2}{9})$

Using DSW and some variations...

"Complete" DSW and using: $(M_5R)^2 = \Delta(\Delta-4) - J$

EFC, D.li and HBF PLB B760 (2016)

"Truncated" DSW using: $(M_5R)^2 = \Delta(\Delta-4)$

EFC and HBF PLB B753 (2016)

"Anomalous" DSW using: $(M_5R)^2 = \Delta(\Delta-4)$

EFC, D. li and HBF EPJC (2016)

Results are in agreement with those found in literature

Last words....

"Anomalous" DSW: model with anomalous dimension contributions coming from two different QCD beta functions cases.

- Beta function with a linear IR asymptotic behaviour:

$$\beta(\lambda) = -\frac{b_0\lambda^2}{1+b_1\lambda}; \quad \text{for} \quad b_0, b_1 > 0.$$

$$\lambda \equiv N_C g_{\text{YM}}^2$$

- Beta function with an IR fixed point at finite coupling:

$$\beta(\lambda) = -b_0\lambda^2\left[1-\frac{\lambda}{\lambda_*}\right]; \quad \text{for} \quad \lambda_* > 0.$$

Even spin

$$\mathcal{O}_4 = Tr(F^2) = Tr(F^{\mu\nu}F_{\mu\nu})$$

$$\mathcal{O}_{4+J} = FD_{\{\mu 1 \ldots}D_{\mu J\}}F, \quad \Longrightarrow \quad \Delta = 4 + J$$

$$R^2M_5^2 = \left[4+J+\beta'(\lambda)-\frac{2}{\lambda}\beta(\lambda)\right] \times \left[J+\beta'(\lambda)-\frac{2}{\lambda}\beta(\lambda)\right]; \quad (\text{even } J).$$

Odd spin

$$\mathcal{O}_6 = SymTr\left(\tilde{F}_{\mu\nu}F^2\right)$$

$$\mathcal{O}_{6+J} = SymTr\left(\tilde{F}_{\mu\nu}FD_{\{\mu 1 \ldots}D_{\mu J\}}F\right) \quad \Longrightarrow \quad \Delta = 6 + J$$

$$R^2M_5^2 = \left[6+J+\beta'(\lambda)-\frac{2}{\lambda}\beta(\lambda)\right] \times \left[2+J+\beta'(\lambda)-\frac{2}{\lambda}\beta(\lambda)\right]; \quad (\text{odd } J).$$

Last words....

"Anomalous" DSW: model with anomalous dimension contributions coming from two different QCD beta functions cases.

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$$\mathcal{O}_6 = SymTr\left(\tilde{F}_{\mu\nu}F^2\right)$$

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$$R^2M_5^2 = \left[6+J+\beta'(\lambda)-\frac{2}{\lambda}\beta(\lambda)\right] \times \left[2+J+\beta'(\lambda)-\frac{2}{\lambda}\beta(\lambda)\right]; \quad (\text{odd } J).$$

Muchas gracias!

Back up Slides
Results in “trucated” DSW

Masses expressed in GeV for the glueball states J^{PC} of the lightest scalar glueball and its radial excitations from the modified softwall model using Eq. (32) for $k =$ 0.2, 0.85 and 1 GeV^2.

	Glueball states J^{PC}				k
	0^{++}	0^{++*}	0^{++**}	0^{++***}	
n	0	1	2	3	
m_n	0.89	1.26	1.55	1.79	0.20
m_n	1.84	2.61	3.19	3.69	0.85
m_n	2.00	2.83	3.46	4.00	1.00

Masses expressed in GeV for the glueball states J^{PC} with even J from the original SW using Eq. (14) with $k = 1$ and 2 GeV^2 and from the modified SW using Eq. (33) with $k = 0.2$ GeV^2.

	Glueball states J^{PC}						k
	0^{++}	2^{++}	4^{++}	6^{++}	8^{++}	10^{++}	
Masses	2.83	3.46	4.00	4.47	4.90	5.29	1.00
Masses	4.00	4.90	5.67	6.32	6.93	7.48	2.00
Masses	0.89	2.19	3.30	4.38	5.44	6.49	0.20

$$J(m^2) = (0.23 \pm 0.02)m^2 + (0.82 \pm 0.51)\,.$$

Masses expressed in GeV for the glueball states J^{PC} with odd J from SW using eq. (18) and $k = 1$ and 2 GeV^2 and from the modified SW using eq. (34) and $k =$ 0.2 GeV^2.

	Glueball states J^{PC}						k
	1^{--}	3^{--}	5^{--}	7^{--}	9^{--}	11^{--}	
Masses	3.74	4.24	4.69	5.10	5.48	5.83	1.00
Masses	5.29	6.00	6.63	7.21	7.75	8.24	2.00
Masses	2.82	3.94	5.03	6.11	7.19	8.26	0.20

$$J(m^2) = (0.18 \pm 0.01)m^2 + (0.02 \pm 0.40)$$

Back up Slides
Results in “complete” DSW

Masses m_n expressed in GeV for the glueball states J^{PC} with even J as the eigenstates of Eq. (9) with the potential (12) for $k = 0.10$ GeV^2.

	Glueball states J^{PC}						k
	0^{++}	2^{++}	4^{++}	6^{++}	8^{++}	10^{++}	
m_n	0.51	2.03	3.23	4.40	5.56	6.71	0.10

$$J(m^2) \approx (0.72 \pm 0.49) + (0.25 \pm 0.02)m^2$$

Masses m_n expressed in GeV for the glueball states J^{PC} with odd J solving Eq. (9) with the potential (12) for $k = 0.10$ GeV^2.

	Glueball states J^{PC}						k
	1^{--}	3^{--}	5^{--}	7^{--}	9^{--}	11^{--}	
m_n	2.77	3.91	5.05	6.19	7.33	8.47	0.10

$$J(m^2) \approx (0.20 \pm 0.43) + (0.17 \pm 0.01)m^2$$

Back up Slides
Results in "anomalous" DSW

Table 4 Masses (GeV) for the glueball states J^{PC} with even and odd J with $P = C = \pm 1$ calculated from the anomalous dynamical soft-wall model, Eqs. (56) and (59), and the beta function with an IR fixed point at finite coupling, (37), using five sets of parameters k (GeV^2), λ_0 and λ_* (dimensionless)

Set	Parameters			Glueball states J^{PC}											
	k	λ_0	λ_*	0^{++}	2^{++}	4^{++}	6^{++}	8^{++}	10^{++}	1^{--}	3^{--}	5^{--}	7^{--}	9^{--}	11^{--}
1	0.16	18.5	350	1.69	3.28	4.76	6.23	7.67	9.12	4.02	5.50	6.95	8.40	9.84	10.00
2	0.09	18.5	350	1.62	2.84	4.00	5.14	6.26	7.37	3.42	4.57	5.70	6.82	7.93	9.04
3	0.04	18.5	350	1.56	2.52	3.43	4.32	5.19	6.05	2.98	3.88	4.76	5.62	6.48	7.32
4	0.09	10.5	350	0.79	2.13	3.28	4.39	5.48	6.57	2.72	3.84	4.94	6.03	7.11	8.19
5	0.09	18.5	250	1.64	2.86	4.02	5.16	6.28	7.39	3.44	4.59	5.72	6.84	7.95	9.05

Table 5 Regge trajectories obtained for both pomeron and odderon from the anomalous soft-wall model with dynamical corrections, Eqs. (56) and (59), using the beta function with an IR fixed point at finite coupling, (37), for the sets of parameters presented in Table 4. The errors come from the choice of a linear fit

Set	Pomeron	Odderon
1	$J \approx (0.6 \pm 0.5) + (0.12 \pm 0.01)\,m^2$	$J \approx (-0.1 \pm 0.4) + (0.10 \pm 0.01)\,m^2$
2	$J \approx (0.4 \pm 0.5) + (0.19 \pm 0.02)\,m^2$	$J \approx (-0.4 \pm 0.4) + (0.15 \pm 0.01)\,m^2$
3	$J \approx (0.1 \pm 0.5) + (0.28 \pm 0.02)\,m^2$	$J \approx (-0.8 \pm 0.4) + (0.24 \pm 0.02)\,m^2$
4	$J \approx (0.9 \pm 0.5) + (0.23 \pm 0.02)\,m^2$	$J \approx (0.1 \pm 0.4) + (0.18 \pm 0.01)m^2$
5	$J \approx (0.4 \pm 0.5) + (0.19 \pm 0.02)\,m^2$	$J \approx (-0.4 \pm 0.4) + (0.15 \pm 0.01)\,m^2$