GUASA 2015 Cosmology
CMB anisotropies and polarization

Elena D'Onghia

1 CMB anistropies, continued

We continue discussion of the temperature fluctuations in the CMB.

The power spectrum is again shown in Figure 1, which shows a compilation of the CMB anisotropies measured by a large number of experiments, with the best fit ΛCDM model overplotted.

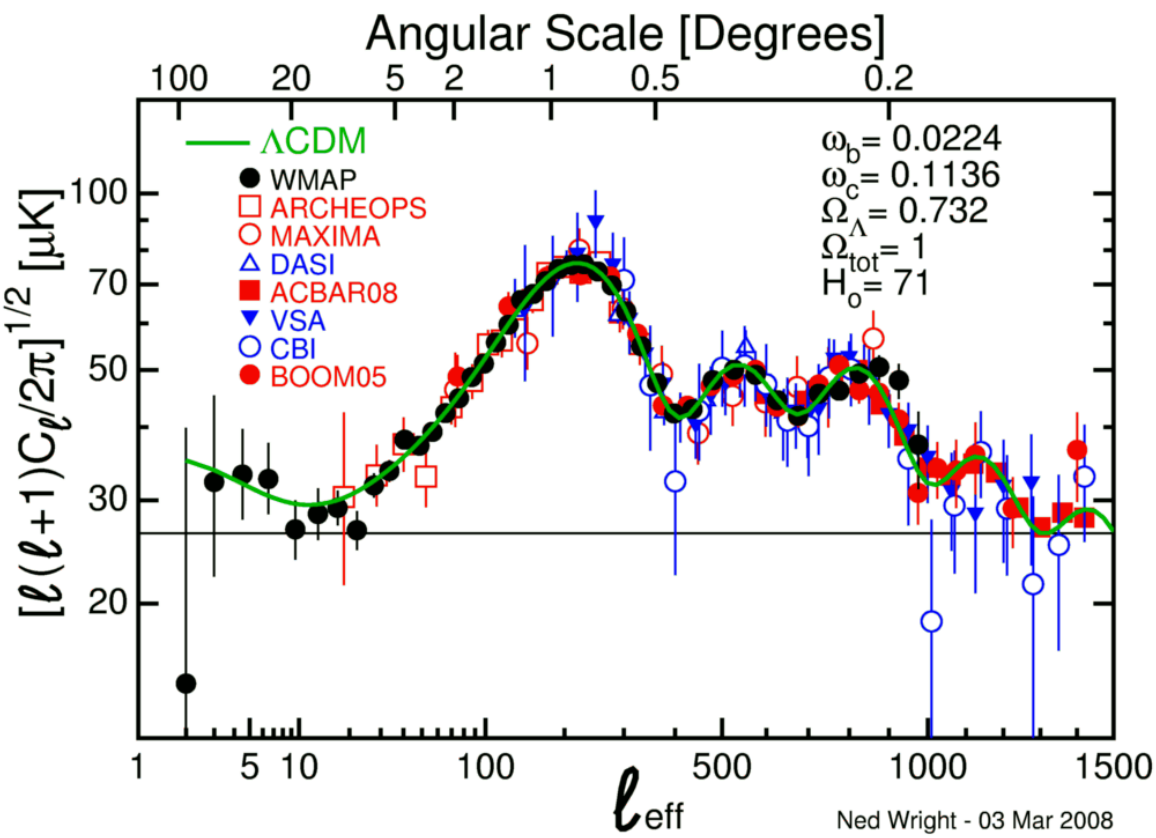

Figure 1: A compilation of CMB anisotropies measured by a number of experiments, assembled by Ned Wright (http://www.astro.ucla.edu/∼wright/CMB-DT.html). $\omega_b = \Omega_b h^2$, and $\omega_c = \Omega_{\rm CDM} h^2$.

1.1 Large scale fluctuations: the Sachs-Wolfe effect

- The large-scale fluctuations with angular size $\theta > \theta_s$ arise from the gravitational effect of primordial density fluctuations in the distribution of nonbaryonic dark matter. The energy density of nonbaryonic dark matter at time of last scattering was

$$u_{\rm dm}(z_{\rm ls}) = \Omega_{\rm dm,0} u_{c,0} (1 + z_{\rm ls})^3 \quad (1)$$

  Putting in numbers, this is

$$u_{\rm dm} \approx (0.26)(5200 \text{ MeV m}^{-3})(1101)^3 \approx 1.8 \times 10^{12} \text{ MeV m}^{-3} \quad (2)$$

- Density of baryonic matter at time of last scattering was

$$u_b(z_{\rm ls}) = \Omega_{\rm b,0} u_{c,0}(1+z_{\rm ls})^3 \approx 2.8 \times 10^{11} \text{ MeV m}^{-3} \tag{3}$$

- Density of photons at time of last scattering, since $u_\gamma \propto a^{-4} \propto (1+z)^4$, was

$$u_\gamma(z_{\rm ls}) = \Omega_{\gamma,0} u_{c,0}(1+z_{\rm ls})^4 \approx 3.8 \times 10^{11} \text{ MeV m}^{-3} \tag{4}$$

- So, at time of last scattering, the energy densities are roughly in the ratio 6.4 : 1.4 : 1, with $u_{\rm dm} > u_\gamma > u_b$, and nonbaryonic dark matter dominates the energy density and gravitational potential.

- If the density of nonbaryonic dark matter isn't perfectly homogeneous and varies as a function of position, we can write the energy density as

$$u(\vec{r}) = \bar{u} + du(\vec{r}) \tag{5}$$

  where $\bar{u}$ is the spatially averaged energy density of the nonbaryonic dark matter and du is the local deviation from the mean.

- In the Newtonian approximation, the spatially varying component of the energy density δu gives rise to a spatially varying gravitational potential $\delta\Phi$ as described by Poisson's equation:

$$\nabla^2(\delta\Phi) = \frac{4\pi G}{c^2}\delta u. \tag{6}$$

- This spatial variation in the gravitational potential affects the spectrum of the CMB. A photon that is at a minimum in the potential at the time of last scattering will lose energy and become redshifted as it climbs out of the potential well, and a photon that happens to be at a potential maximum will gain energy and become blueshifted.

  - Gravitational redshift: $\Delta\lambda/\lambda \approx \delta\Phi/c^2$ in the classical, small $\delta\Phi$ limit

- A detailed general relativistic calculation gives

$$\frac{\delta T}{T} = \frac{1}{3}\frac{\delta\Phi}{c^2}. \tag{7}$$

  So the temperature fluctuations on large angular scales (as seen by COBE for example) of $\theta > \theta_s \approx 1^\circ$ give us a map of the potential fluctuations $\delta\Phi$ at the time of last scattering. This is called the **Sachs-Wolfe effect**, after Sachs and Wolfe who first calculated it in 1967.

1.2 Cosmological parameters

The CMB power spectrum is sensitive to a wide range of cosmological parameters, often in very subtle ways. As we have seen, the simplest of these is the location of the first peak, which tells us that the universe is flat. Once we make that assumption, three of the easiest parameters to measure are the baryon density Ω_b, the matter density Ω_m, and the Hubble constant h. Actually, as shown in Figure 2, the CMB anisotropies are most sensitive to combinations of these parameters.

Here is a general description of the sensitivity of the anisotropies to these cosmological parameters:

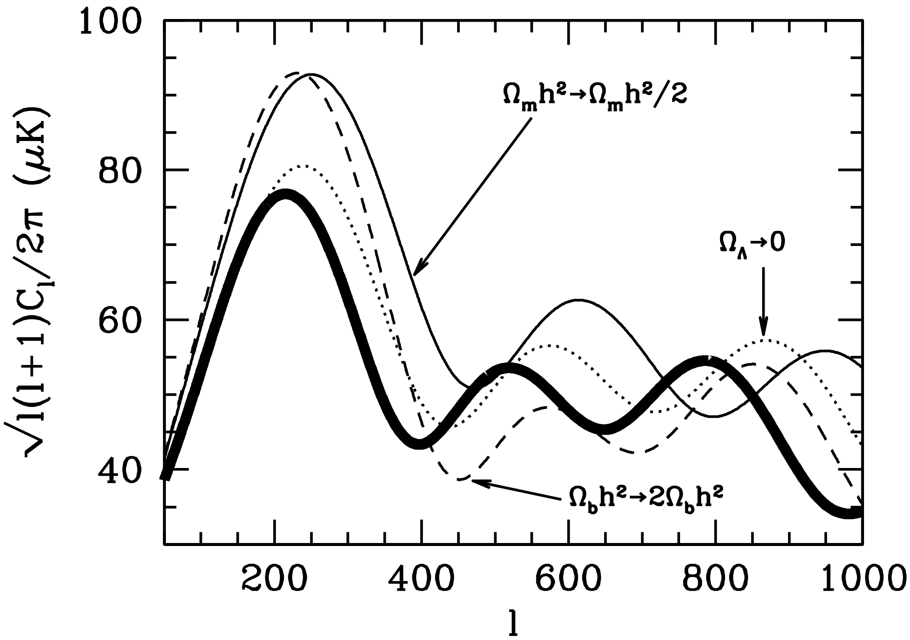

Figure 2: Changes in the spectrum of CMB anisotropies as the baryon density, matter density and cosmological constant vary. The dark solid curve shows a base model with $\Omega_m h^2 = 0.16$, $\Omega_b h^2 = 0.021$ and $\Omega_\Lambda = 0.7$. The total density is set to the critical density. From Dodelson, *Modern Cosmology*.

- Baryon density $\Omega_b h^2$. The sound speed goes down as more baryons are added. The frequency of oscillation thus becomes smaller as the baryon density goes up. A reduced frequency accentuates the effectiveness of the driving force, making the oscillation more asymmetric. The result is that the height of the second peak is much smaller than the height of the first peak when the baryon density is high.

- Matter density $\Omega_m h^2$. The spectrum is sensitive to the ratio of matter to radiation, particularly at smaller angular scales since fluctuations corresponding to higher peaks entered the sound horizon at earlier times, during radiation domination. The presence of radiation affects the gravitational potential and effectively boosts the oscillations. The similar heights of the second and third peaks tell us that most of the matter in the universe is non-baryonic, and that dark matter dominated the energy density at recombination.

- Cosmological constant Ω_Λ. The cosmological constant is a late time effect, so the only impression it leaves on the CMB relates to the way physical scales project onto angular scales; i.e. Ω_Λ changes the distance to the last scattering surface, so the curves simply shift horizontally if Ω_Λ changes.

The effects of these changes on the power spectrum are also shown in Figure 3. Figure 4 shows the combined constraints on cosmological parameters from the CMB, the Type Ia supernova measurements, and measurements of the matter density from galaxy clusters.

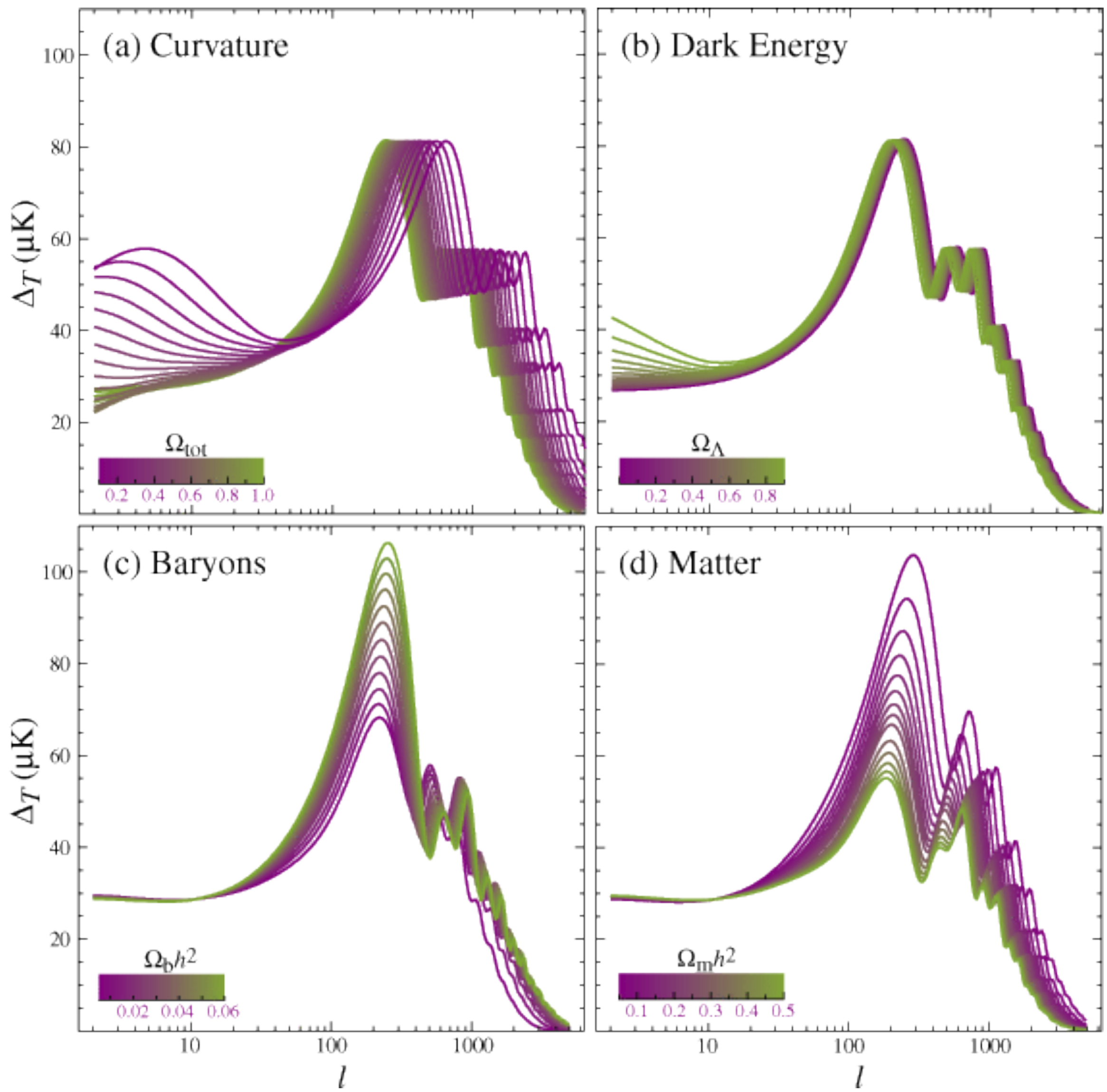

Figure 3: Sensitivity of the CMB power spectrum to changes in curvature, dark energy content, and the density of baryons and matter. Curvature shifts the spectrum horizontally because it affects the mapping between physical and angular scales. Dark energy changes the distance to the last scattering surface, so it also produces a horizontal shift. The relative densities of baryons and dark matter have a strong effect on the relative heights of the peaks. The parameters are varied around a fiducial model with $\Omega_{\rm tot} = 1$, $\Omega_\Lambda = 0.65$, $\Omega_b h^2 = 0.02$, and $\Omega_m h^2 = 0.147$. From Hu & Dodelson, *Annu. Rev. Astron. and Astrophys.* 40, 171 (2002).

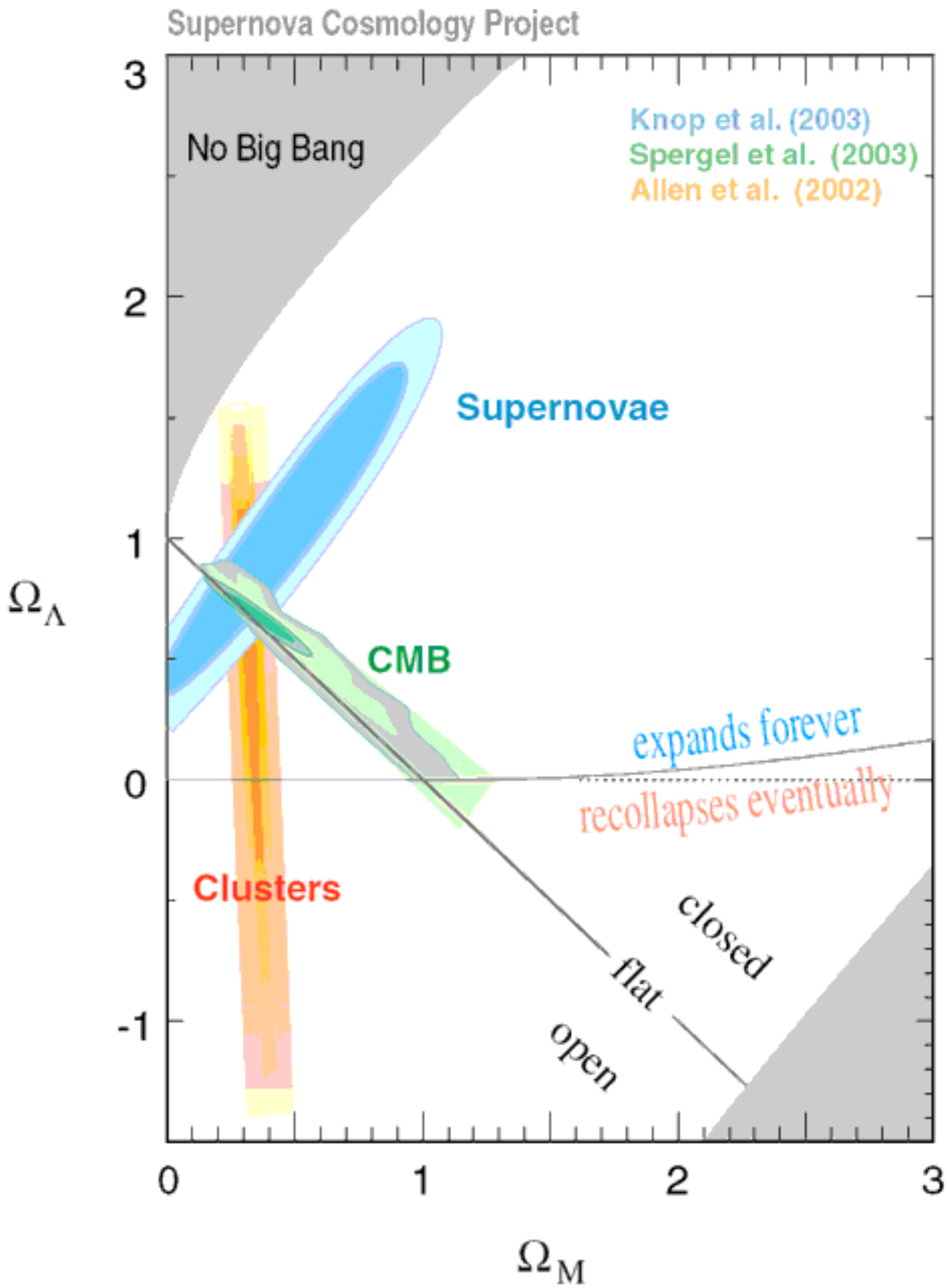

Figure 4: Combined constraints on cosmological parameters from the CMB, the Type Ia supernova measurements, and measurements of the matter density from galaxy clusters.

2 CMB polarization

The CMB anisotropies contain additional important information: they are (partially) polarized, at about the $\sim 10\%$ level.

2.1 Why is the CMB polarized?

Polarization of the CMB comes from Thomson scattering of CMB photons by the free electrons in the photon-baryon fluid. The process is shown in Figures 5 and 6.

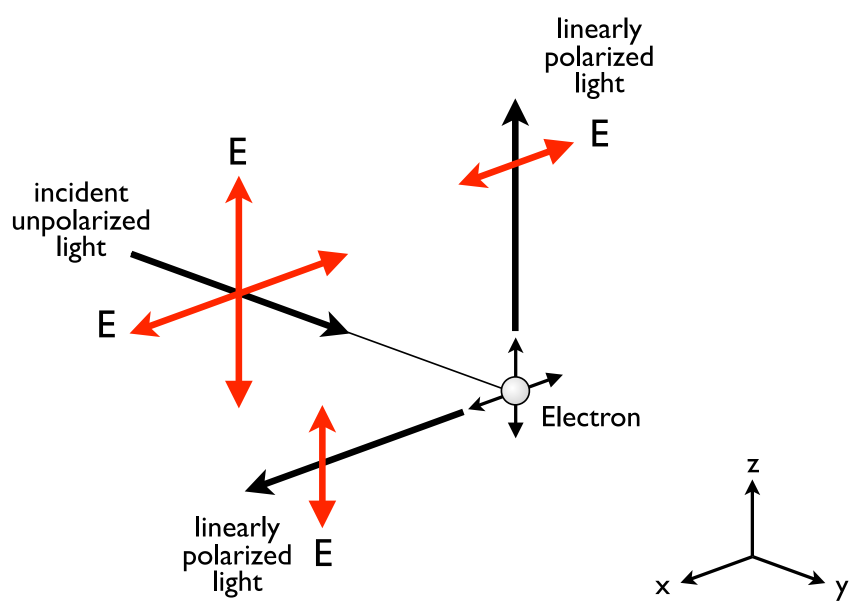

Figure 5: Incident unpolarized light from one direction produces linearly polarized light when scattered. If the radiation field is isotropic and incident from all directions there will be no net polarization.

Figure 5 shows how the electric field of incident unpolarized radiation (in the $+y$ direction) causes an electron to vibrate. This produces scattered light in the $+x$ and $+y$ direction that is linearly polarized. However, if the radiation field is isotropic and coming at the electron from all directions, there will be no net polarization.

This is where the temperature anisotropies become important: to produce partial polarization of the CMB photons, we need a radiation field more intense along one axis than along another. This is shown in Figure 6, where the incident radiation in the $+y$ direction is more intense than in the $+x$ direction. This radiation has a *quadrupole moment* (because the poles of anisotropy are separated by 90°). The result is that the radiation scattered along the $+z$ axis is partially polarized in the $+x$ direction.

If the photons are scattered frequently, their directions quickly become randomized and the quadrupole moment can't be maintained. This means that the polarization must be produced in a short time interval toward the end of recombination, when there are still free electrons but photons don't scatter enough to become randomized. This is why only a small fraction of the CMB radiation is polarized.

How do we get a local quadrupole moment in the CMB?

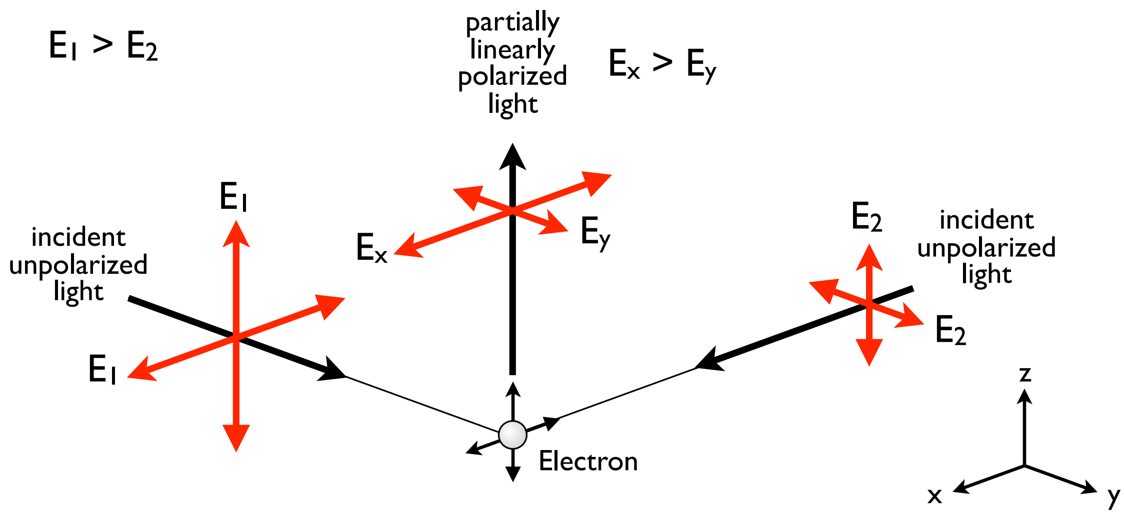

Figure 6: Incident unpolarized light of different intensities ($E_1 > E_2$) produces partially linearly polarized light ($E_x > E_y$).

- *Temperature fluctuations*: At radiation decoupling when photons last scattered off of electrons, there was temperature inhomogeneity, as we have already discussed. When recombination proceeded to the point where photons from hot and cold regions could meet to be scattered by the same electron, the scattered radiation was polarized. For the example shown in Figure 6, the higher intensity light comes from a hotter region than the lower intensity light, resulting in polarization.

  Electrons at different locations would produce different polarization orientations and magnitudes. As observed today, the CMB polarization varies across the sky. The quadrupole anisotropies at decoupling are projected into CMB polarization pattern. Since photons could not diffuse too far, polarization doesn't vary much across very large angular scales.

- *Density fluctuations*: The acoustic oscillations of the photon-baryon fluid produce a velocity gradient within the fluid. Consider the simple situation in which the fluid is moving radially inward toward a local concentration of dark matter (see Figure 7). From the frame of reference of an electron in the field, the neighboring elements of the fluid are all moving toward it, with the fluid in the radial direction approaching more rapidly than the fluid in the transverse direction. The radiation from the radial directions is blueshifted and therefore more intense than the radiation from the transverse directions. This produces the quadrupole moment in the local radiation field that is responsible for partially polarizing the scattered photons.

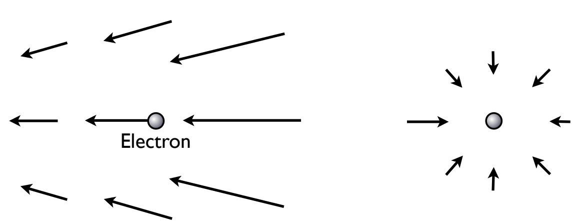

Figure 7: *Left*: The motion of an electron in the photon-baryon fluid as the fluid moves radially inward toward maximum compression. *Right*: The velocity field seen from the rest frame of the electron.

The polarization pattern (the direction of the observed electric field of the CMB photons) is directed radially away from a cooler temperature fluctuation and forms tangential loops around a warmer fluctuation. (Note that in Figure 6 the electric field of the partially polarized light is along the direction of the less intense source of light and perpendicular to the direction of the more intense source.) This pattern is called an **E-mode**. Temperature and density fluctuations generate only E-modes.

- *Gravitational waves*: It is also possible to produce a quadrupole moment through the effect of gravitational radiation. When a gravitational wave passes through the photon-baryon fluid, the space containing the photos is stretched along one axis and compressed along a perpendicular axis. This changes the wavelengths of the photons, producing a quadrupole moment in the CMB. Gravitational waves can produce the E-modes discussed above, but they can also produce **B-modes**, which spiral outward from the temperature fluctuations.

  Inflation predicts primordial gravitational waves, so measurement of the B-mode signal due to gravitational waves would provide a measurement of the energy scale of inflation and place constraints on inflationary theories. The expected signal is very weak and needs to be distinguished from foreground effects that can also produce B-modes.

Any polarization pattern can be decomposed into E-modes and B-modes. E-modes are radial or tangential with no preferred handedness, like an electric field. Like magnetic fields, B-modes do have handedness; note that if reflected across a line going through the center the E-patterns are unchanged, whereas the positive and negative B patterns get interchanged. E-modes have zero curl, and B-modes have zero divergence.

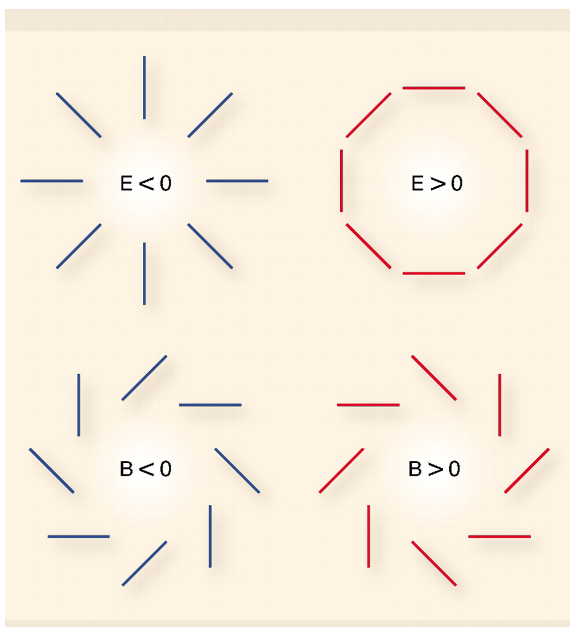

Figure 8: Any polarization pattern can be decomposed into E modes (zero curl, radial and tangential, with no handedness) and B modes (zero divergence, with handedness).

2.2 Measurements of CMB polarization

Observations of the polarization of the CMB carry a lot of information about the conditions at the time of decoupling. We have detected E-modes in the CMB, but not B-modes, which are expected to be much weaker. Measurement requires careful accounting of polarized foregrounds:

- Synchrotron emission produced by cosmic ray electrons orbiting in the Galactic magnetic field can be strongly polarized in the direction perpendicular to the magnetic field.
- Dust. Nonspherical dust grains align their long axes perpendicularly to the Galactic magnetic field. The aligned grains preferentially absorb the component of starlight polarized along their longest axis. Thus, when we observe starlight we see it polarized in the same direction as the magnetic field. These same grains emit thermal radiation preferentially polarized along their longest axis, perpendicular to the Galactic magnetic field.
- So we expect to observe thermal dust emission and synchrotron emission polarized in the same direction, while starlight is polarized perpendicularly to both.
- This foreground polarization needs to be mapped and accounted for in our measurements of the CMB.

We make maps of the E-modes and B-modes (if we were to detect them) and decompose them into spherical harmonics as for the temperature fluctuations. The polarization signals are $\sim$ 1–2 orders of magnitude weaker than the temperature fluctuations because polarized radiation was produced only near the end of recombination. The polarization spectra decline at large angular scales (low l) because photons couldn't diffuse so far before the end of recombination.

We look at correlation functions of the polarization modes in the same way that we look at the correlation function of temperature fluctuations. We also look at the correlation between the polarization and temperature fluctuations. Because of the effect described above, in which the acoustic oscillations are responsible for the quadrupole moment producing the partial polarization, we expect a correlation between the temperature and the E-modes.

The acoustic oscillations have the largest velocity gradients where the wave displacement is at a maximum, but the maximum compression or rarefaction of the medium occurs at the points of zero displacement. The quadrupole moment producing the polarization is due to the velocity gradients in the fluid, while the temperature fluctuations are due to the compression and rarefaction of the medium.

This means that the peaks in the angular power spectrum of the temperature fluctuations should be out of phase with the E-mode peaks; i.e., the first peak in the angular power spectrum (corresponding to maximum compression at the time of last scattering) should correspond to a minimum in the E-mode curve, and that is indeed what we observe (see Figure 9).

The E-mode peaks around an angular scale corresponding to the photon mean free path at decoupling.

E-modes have been detected with WMAP, as shown in Figure 9, and several other experiments. B-modes have not yet been detected, but they are expected to be very faint. The recently launched Planck satellite will improve measurements of the E-modes, and may detect the B-modes as well. A detection of the B-modes would be very important, because the gravitational waves that produce them are expected to originate from the end of inflation. Because of the scattering of photons by free electrons before recombination, we will never be able to see beyond the surface of last scattering with electromagnetic radiation. Observations of gravitational waves, either directly or through their imprint on the polarization of the CMB, are therefore very important in constraining the conditions in the very early universe.

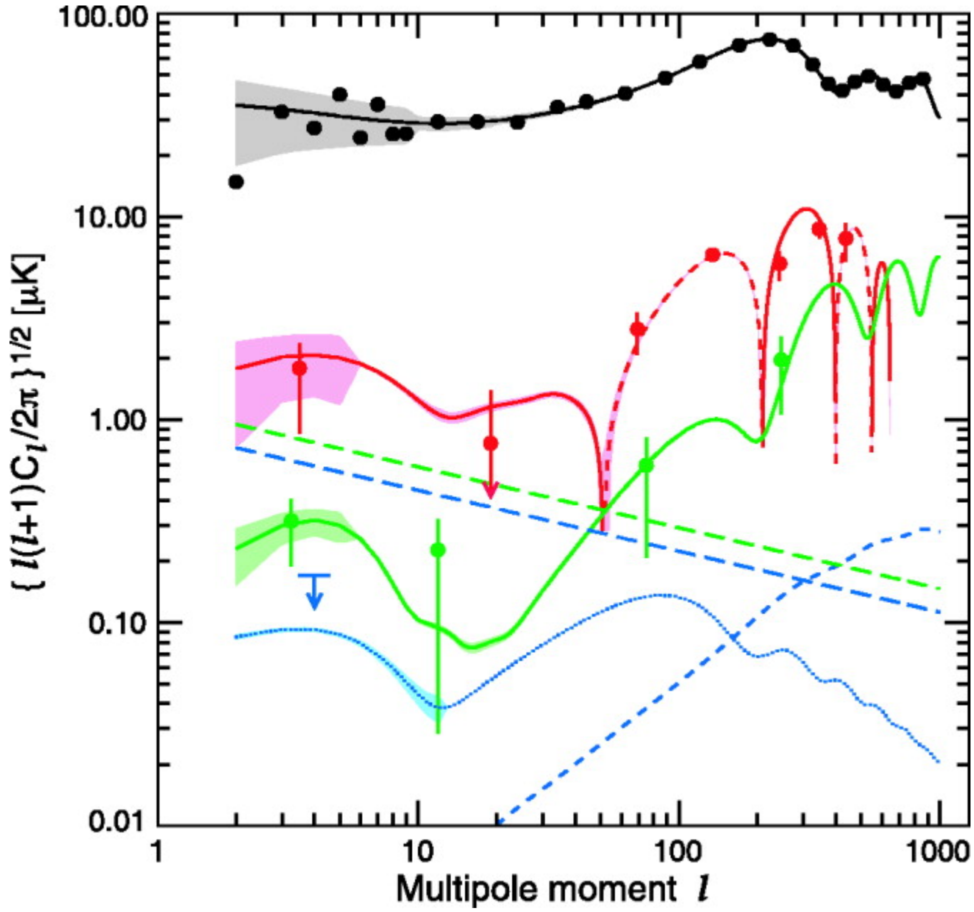

Figure 9: The WMAP measurements of the polarization of the cosmic microwave background. The familiar angular power spectrum of temperature fluctuations is shown in black at the top of the figure. The temperature/E-mode correlation is shown in red, the E-mode polarization in green, and the upper limit on the B-mode by the blue dotted line. Note that the first peak in the temperature power spectrum corresponds to a minimum of the E-modes. Dashed lines show foregrounds: straight lines for synchrotron and dust emission, with E-modes in green and B-modes in blue, and the curved line shows the effect of foreground lensing on B-modes. From Page et al., 2007, *ApJS*, 170, 335.